

Alice in Stretch & SqueezeLand: 12 Cover and Image Relations

August 12, 2012

Chapter Abstract

Alice in
Stretch &
SqueezeLand:
12 Cover
and Image
Relations

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Cover \leftrightarrow Image

Some dynamical system equations exhibit symmetry.

Sometimes the strange attractors they generate also exhibit symmetry.

Symmetry complicates analysis.

There is a simple algorithm for “modding out” symmetry.

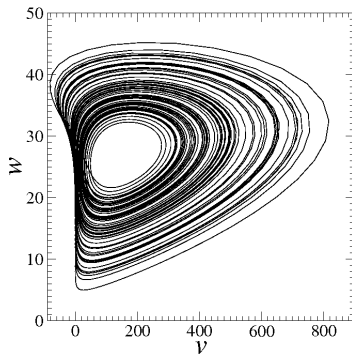
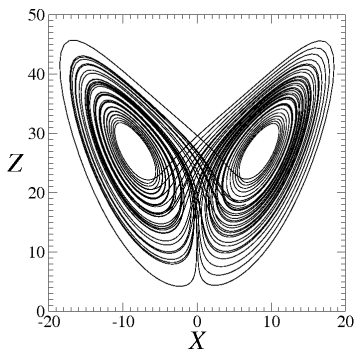
Symmetry can be (re)introduced in a delightful variety of ways.

Covers with the same symmetry differ by topological indices.

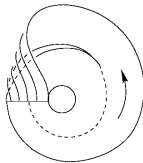
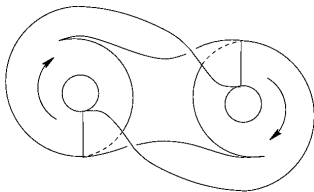
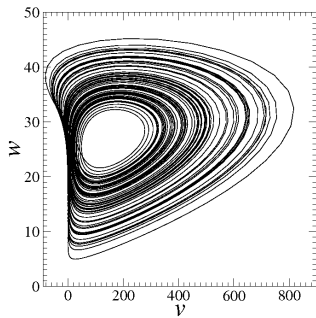
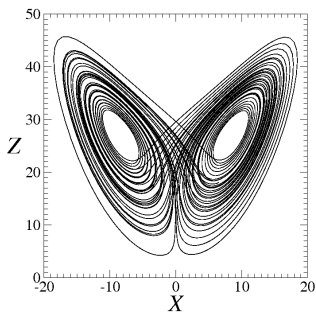
Modding Out a Rotation Symmetry

Modding Out a Rotation Symmetry

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \rightarrow \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \operatorname{Re} (X + iY)^2 \\ \operatorname{Im} (X + iY)^2 \\ Z \end{pmatrix}$$



Lorenz Attractor and Its Image



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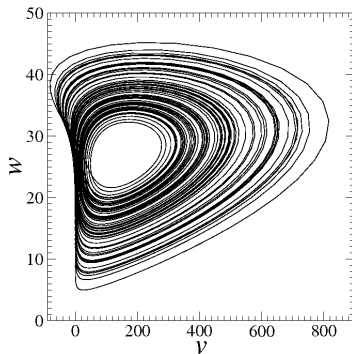
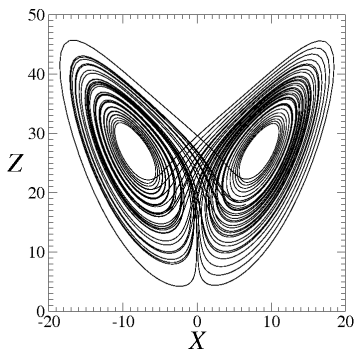
Covers-10

Covers-11

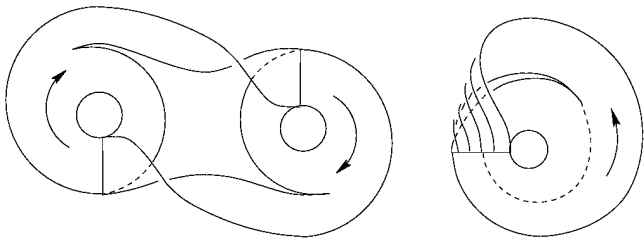
Lifting an Attractor: Cover-Image Relations

Creating a Cover with Symmetry

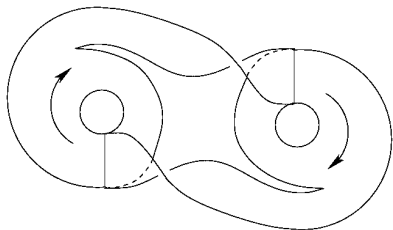
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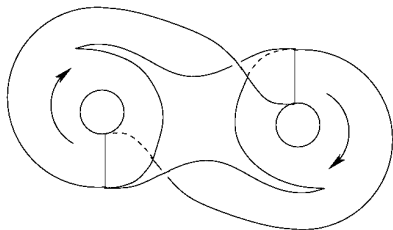
Cover-Image Branched Manifolds



Two Two-fold Lifts Different Symmetry

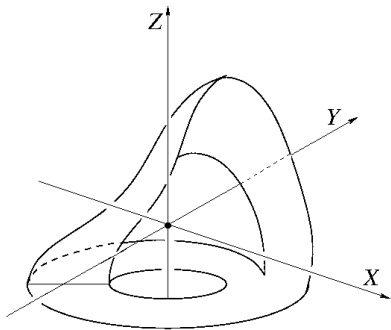


**Rotation
Symmetry**

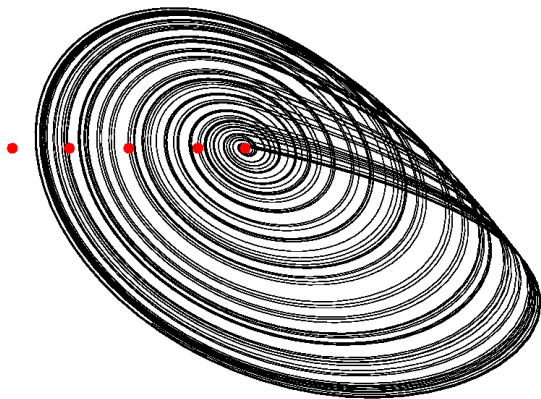


**Inversion
Symmetry**

Topological Index: Choose Group Choose Rotation Axis (Singular Set)



Different Rotation Axes Produce Different (Nonisotopic) Lifts



Nonisotopic Locally Diffeomorphic Lifts

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(a) $\mu = 0.0$



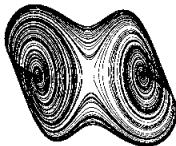
(c) $\mu = -2.083$



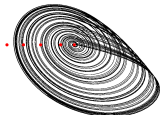
(e) $\mu = -4.166$



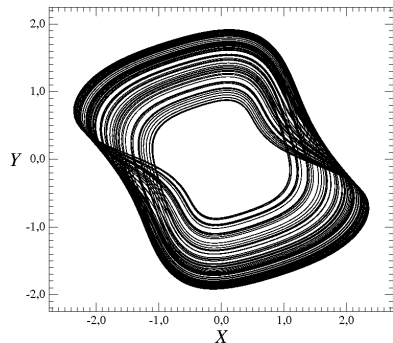
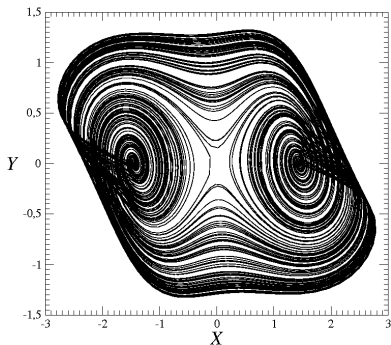
(b) $\mu = -0.84548$



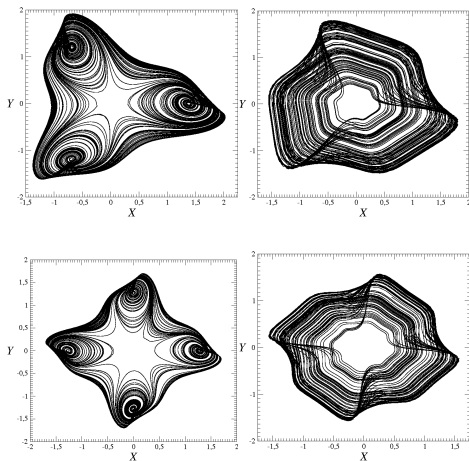
(d) $\mu = -3.14674$



Two Two-fold Covers Same Symmetry



Three-fold, Four-fold Covers



Two Inequivalent Lifts with V_4 Symmetry

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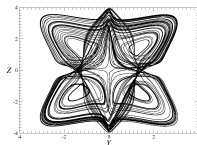
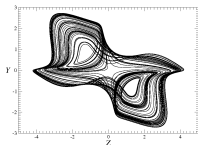
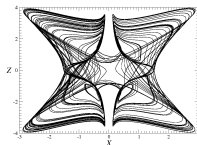
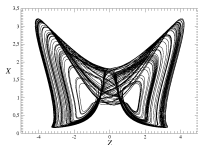
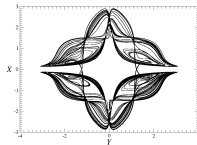
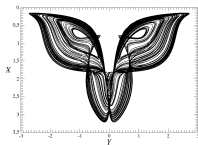
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Algorithm

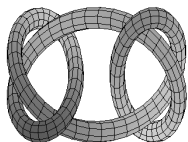
- Construct Invariant Polynomials, Syzygies, Radicals
- Construct Singular Sets
- Determine Topological Indices
- Construct Spectrum of Structurally Stable Covers
- Structurally Unstable Covers Interpolate

Symmetries Due to Symmetry

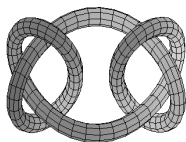
- Schur's Lemmas & Equivariant Dynamics
- Cauchy Riemann Symmetries
- Clebsch-Gordon Symmetries
- Continuations
 - Analytic Continuation
 - Topological Continuation
 - Group Continuation

Covers of a Trefoil Torus

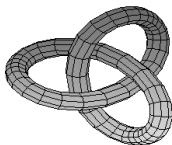
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Granny Knot



Square Knot



Trefoil Knot

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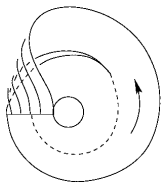
Covers-09

Covers-10

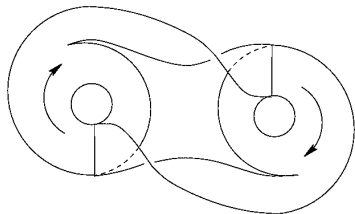
Covers-11

You Can Cover a Cover = Lift a Lift

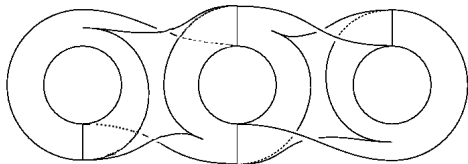
Covers of Covers of Covers



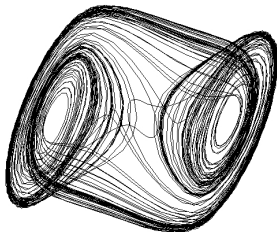
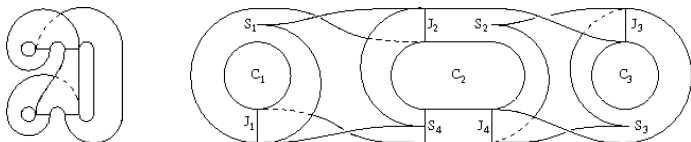
Rossler



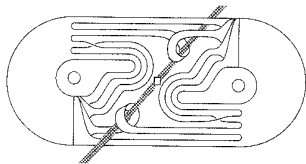
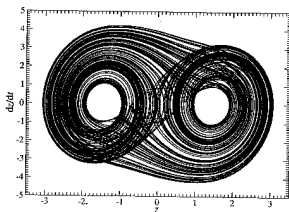
Lorenz



EveryKnot Lives Here



Chua Attractor & Template



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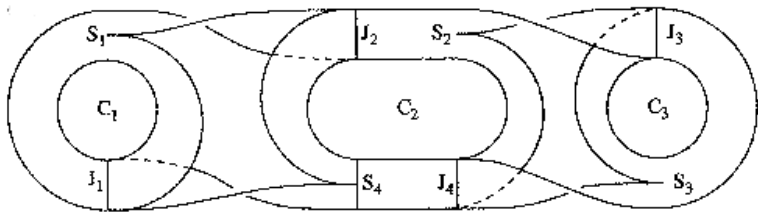
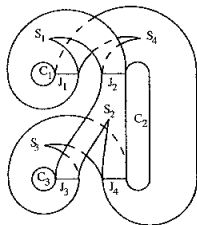
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Universal Template



Local Stuff

Groups:

Local Isomorphisms

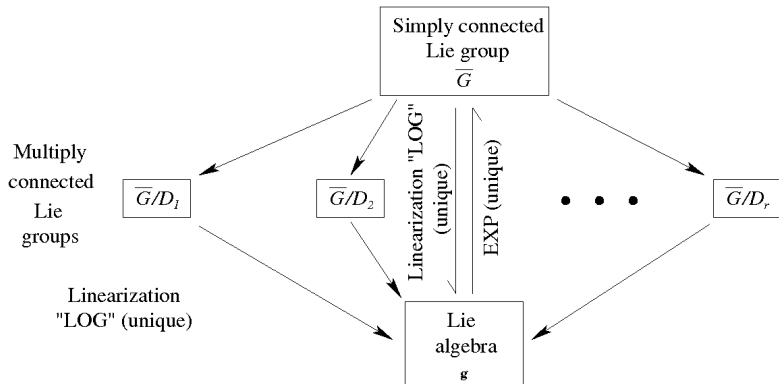
Cartan's Theorem

Dynamical Systems:

Local Diffeomorphisms

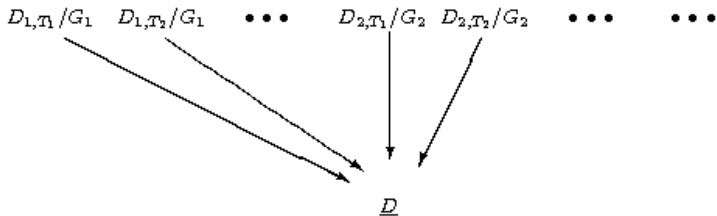
??? Anything Useful ???

Cartan's Theorem for Lie Groups



Universal Image Dynamical System

Locally Diffeomorphic Covers of \underline{D}



\underline{D} : Universal Image Dynamical System

Local Isomorphisms & Diffeomorphisms

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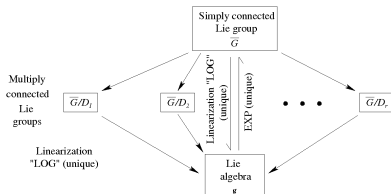
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Local Isomorphisms & Diffeomorphisms

Lie Groups

Local Isomorphisms

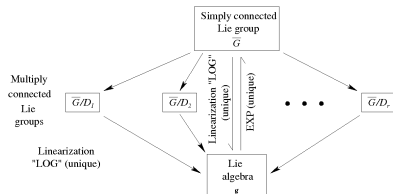


Local Isomorphisms & Diffeomorphisms

Lie Groups

Dynamical Systems

Local Isomorphisms



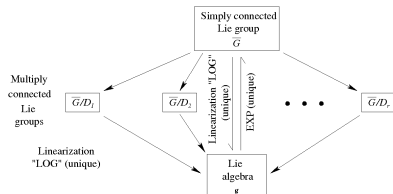
Local Isomorphisms & Diffeomorphisms

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Local Diffeos



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