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August 12, 2012

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Chapter Abstract

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Periodic orbits exist in abundance in a strange attractor.

The problems are: to find them, to determine how they are organized among themselves.

Whatever mechanism exists to create the strange attractor, it simultaneously organizes all the unstable periodic orbits in the attractor in a unique way.

We can classify *mechanisms* by sets of *integers*.

There is an Aufbau Principal for building up strange attractors.

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Chaos

Chaos

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Motion that is

• Deterministic:

$$\frac{dx}{dt} = f(x)$$

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- Recurrent
- Non Periodic
- Sensitive to Initial Conditions

Strange Attractor

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Strange Attractor

The Ω limit set of the flow. There are unstable periodic orbits "in" the strange attractor. They are

- "Abundant"
- Outline the Strange Attractor
- Are the Skeleton of the Strange Attractor

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Skeletons

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Topology of

UPOs Outline Strange Attractors



Figure 5. Left: a chaotic attractor reconstructed from a time series from a chaotic laser; Right: Superposition of 12 periodic orbits of periods from 1 to 10.

Lefranc - Cargese

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Topology of

Organization of UPOs in R³: Gauss Linking Number

$$LN(A,B) = \frac{1}{4\pi} \oint \oint \frac{(\mathbf{r}_A - \mathbf{r}_B) \cdot d\mathbf{r}_A \times d\mathbf{r}_B}{|\mathbf{r}_A - \mathbf{r}_B|^3}$$

Interpretations of LN $\simeq \#$ Mathematicians in World

Linking Numbers

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Linking Number of Two UPOs



Figure 6. Left: two periodic orbits of periods 1 and 4 embedded in a strange attractor; Right: a link of two knots that is equivalent to the pair of periodic orbits up to continuous deformations without crossings.

Lefranc - Cargese

Mechanisms for Generating Chaos

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Topology of

Stretching and Folding



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Mechanisms for Generating Chaos



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Motion of Blobs in Phase Space

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Stretching — Squeezing



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Topology of

Birman - Williams Projection

Collapse Along the Stable Manifold

Identify x and y if

 $\lim_{t\to\infty}|x(t)-y(t)|\to 0$



FLOW ----

Fundamental Theorem

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Birman - Williams Theorem

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Then:

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Certain Assumptions

Then:

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Fundamental Theorem

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Then:

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Birman-Williams Theorem

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Assumptions, B-W Theorem

A flow $\Phi_t(x)$

• on \mathbb{R}^n is dissipative, $\underline{n=3}$, so that $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 < 0, \qquad \lambda_1 + \lambda_2 + \lambda_3 < 0$

• Generates a <u>hyperbolic</u> strange attractor \mathcal{SA}

IMPORTANT: The underlined assumptions can be relaxed.

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Conclusions, B-W Theorem

• The projection maps the strange attractor SA onto a 2-dimensional branched manifold \mathcal{BM} and the flow $\Phi_t(x)$ on SA to a semiflow $\overline{\Phi}(x)_t$ on \mathcal{BM} .

• UPOs of $\Phi_t(x)$ on SA are in 1-1 correspondence with UPOs of $\overline{\Phi}(x)_t$ on BM. Moreover, every link of UPOs of $(\Phi_t(x), SA)$ is isotopic to the correspond link of UPOs of $(\overline{\Phi}(x)_t, BM)$.

Remark: "One of the few theorems useful to experimentalists."

A Very Common Mechanism

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Topology of

Attractor Branched Manifold

Rössler:





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A Mechanism with Symmetry

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Lorenz: Attractor Branched Manifold



Examples of Branched Manifolds

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Topology of

Inequivalent Branched Manifolds





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Ghrist Universal Template

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Topology of

Template Holding All Knot Types



Aufbau Princip for Branched Manifolds

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Any branched manifold can be built up from stretching and squeezing units



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subject to the conditions:

- Outputs to Inputs
- No Free Ends

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Topology of

Rossler System









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Topology of

Poincaré Smiles at Us in R³

- \bullet Determine organization of UPOs \Rightarrow
- \bullet Determine branched manifold \Rightarrow
- \bullet Determine equivalence class of \mathcal{SA}