Alice in Stretch & SqueezeLand: 3 Belousov-Zhabotinskii Chemical Reaction

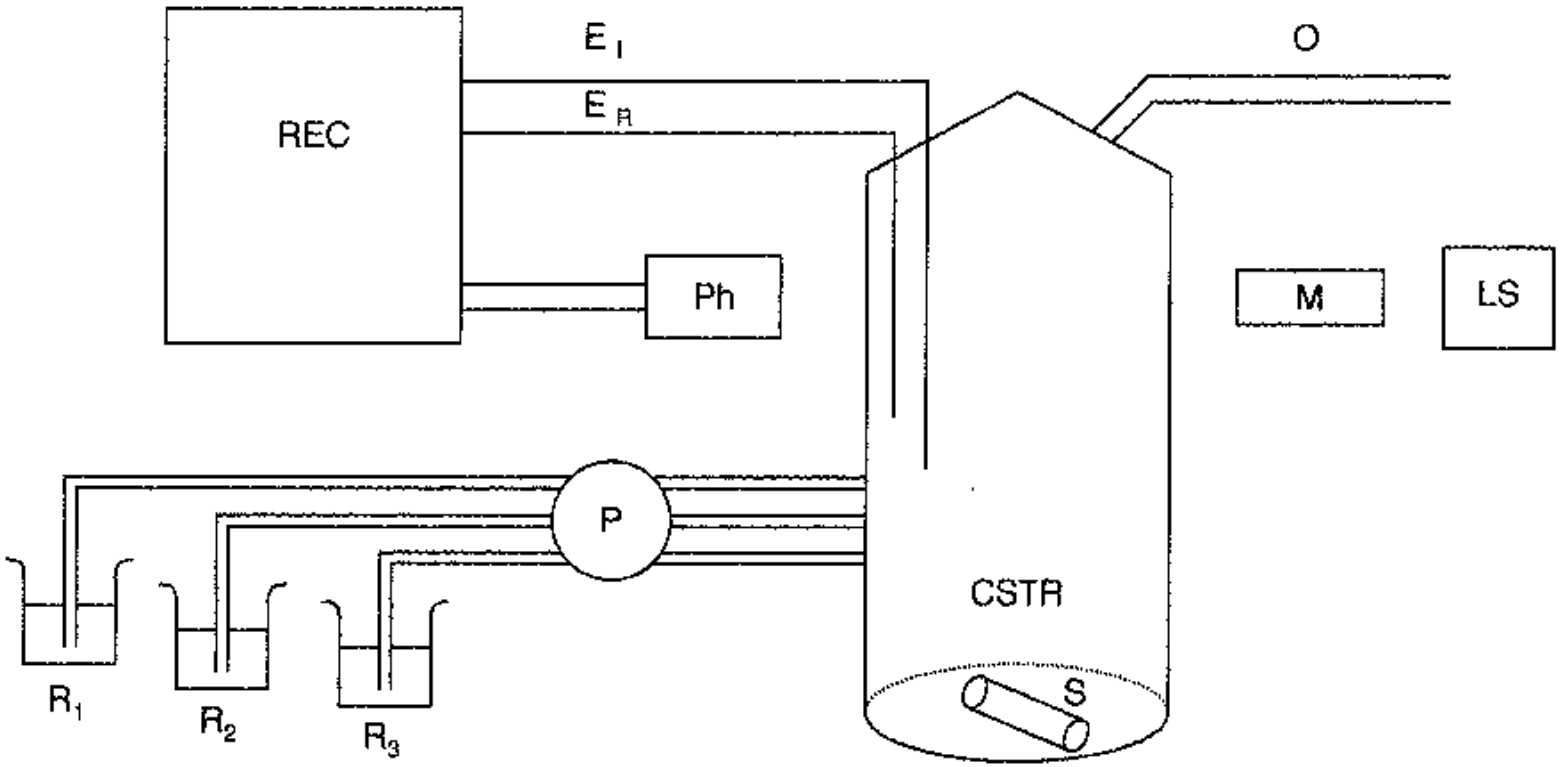
August 12, 2012

Chapter Abstract

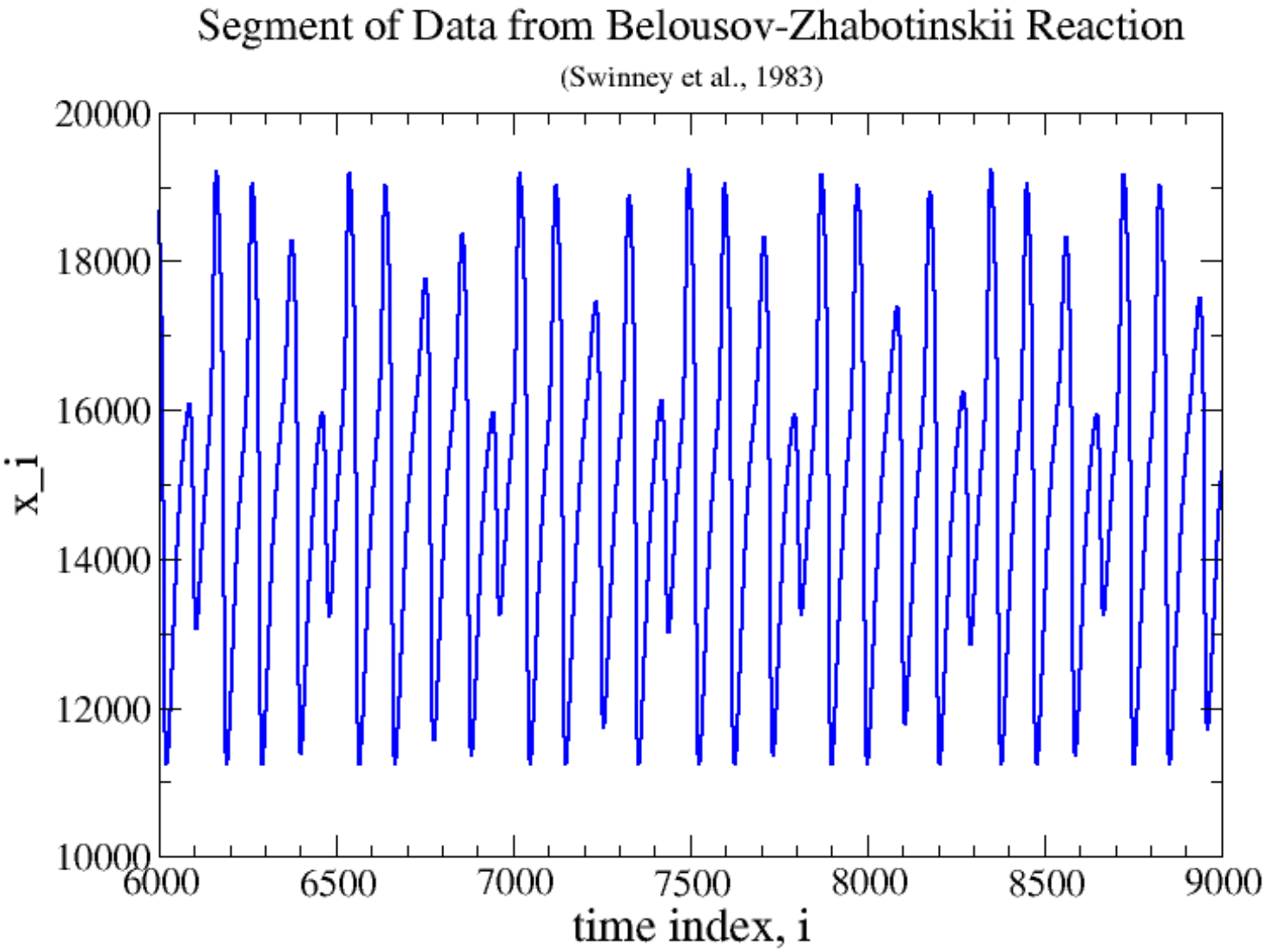
Our first attempts to perform a topological analysis were carried out on intensity data from the Laser with Modulated Losses. This attempt failed.

Our second attempt was carried out on time series data from the Belousov-Zhabotinskii chemical reaction. The data were taken by Swinney's group and consisted of digitized recordings of the log of the Bromine ion concentration.

Belousov-Zhabotinskii Experimental Configuration

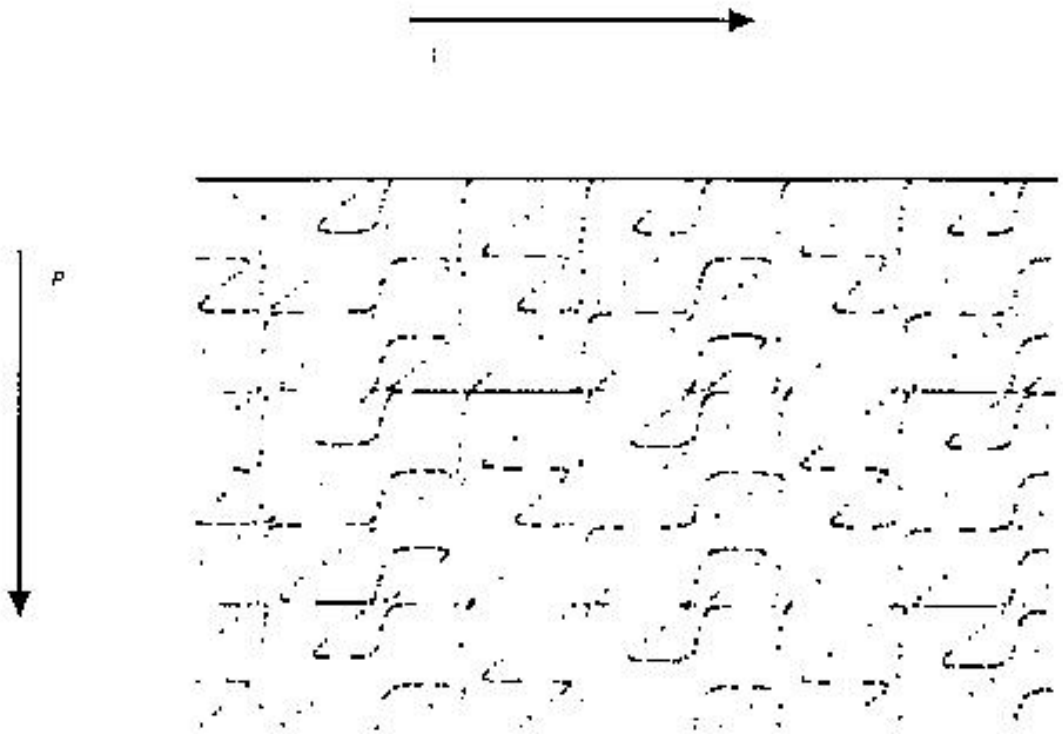


Belousov-Zhabotinskii Experimental Data

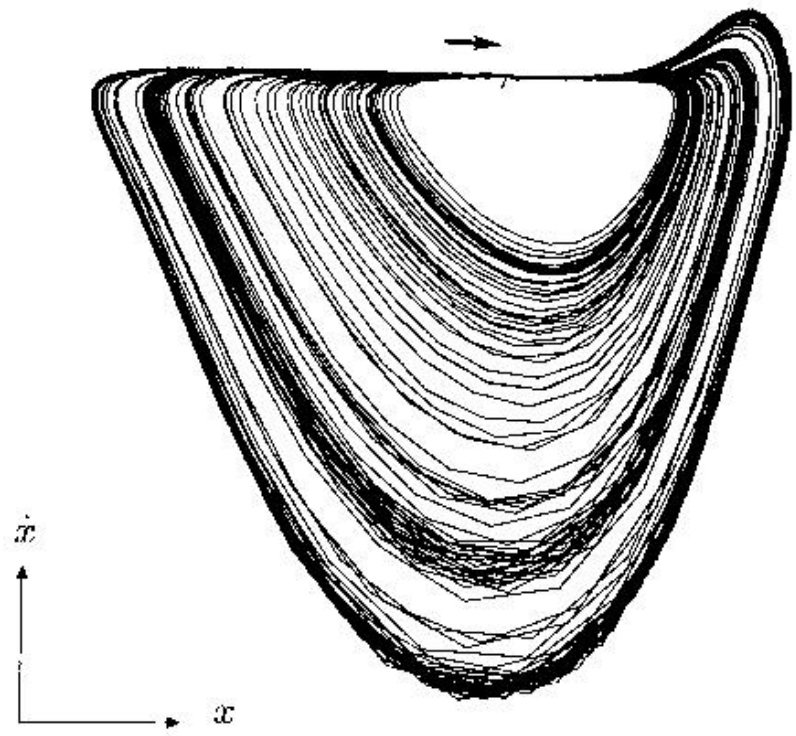


Close Returns Plot

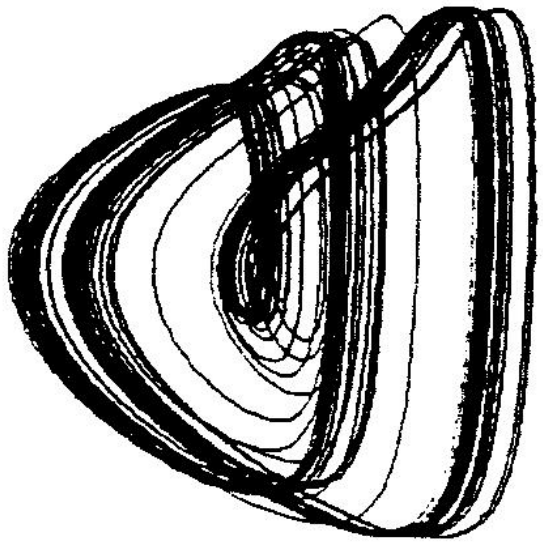
$$|x_i - x_{i+p}| < \epsilon \qquad \text{pixel} \rightarrow \text{black}$$



First Embedding Attempt: $x, \dot{x}, \ddot{x}$

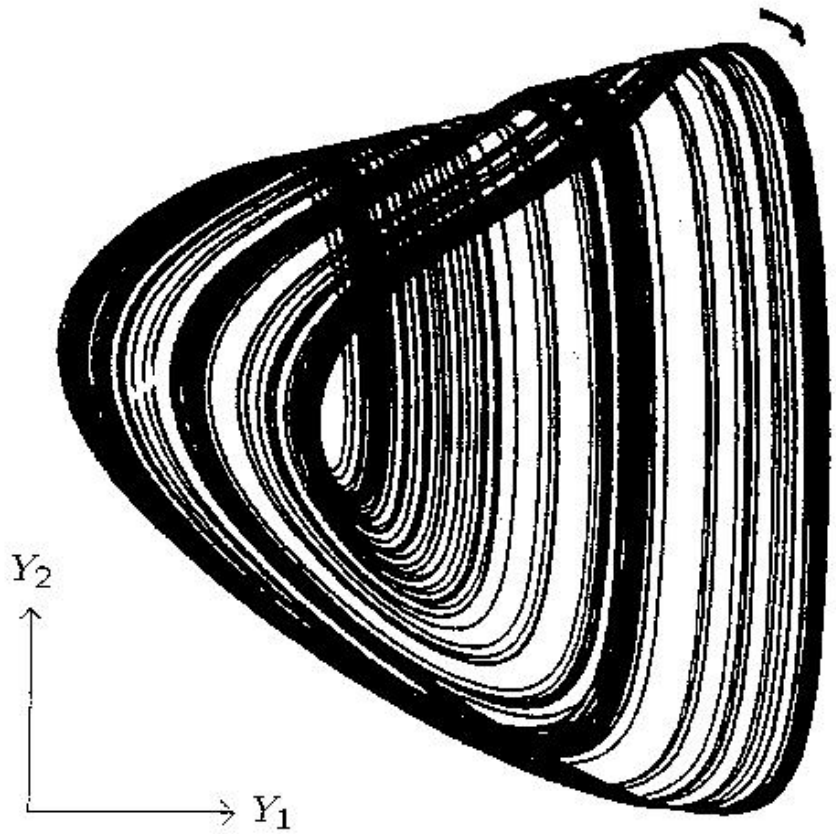


Second Embedding Attempt: $\int x, x, \dot{x}$



Nonstationary!

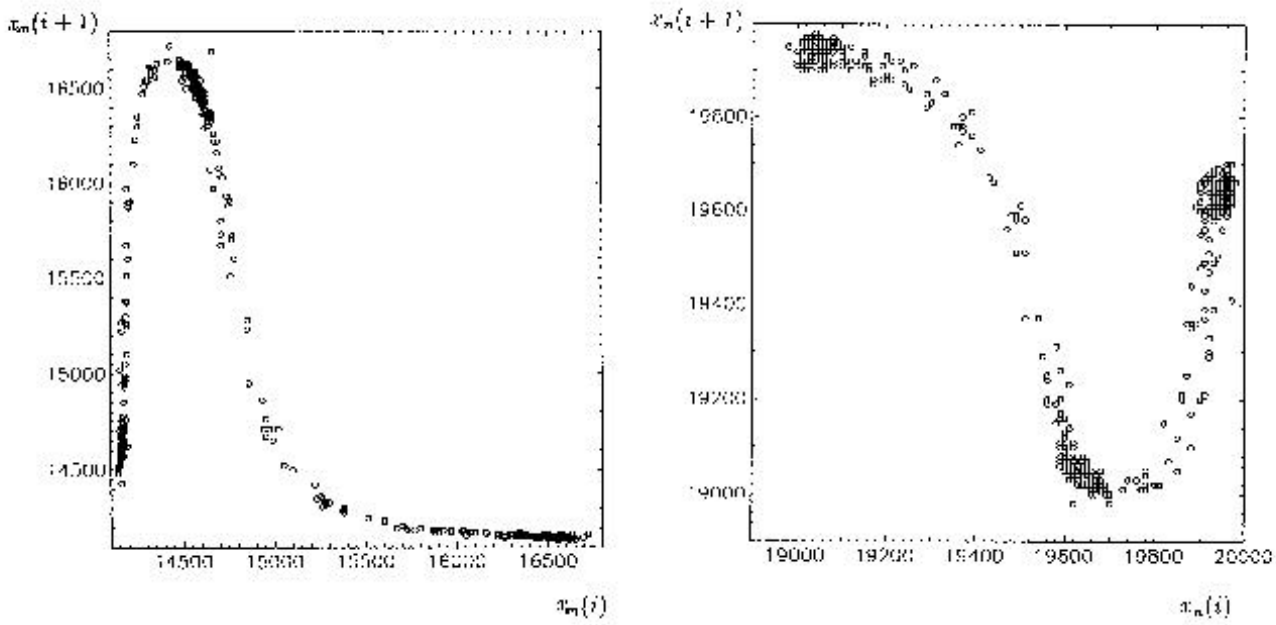
Third embedding attempt: $\int xe^{-t'/\tau}, x, \dot{x}$



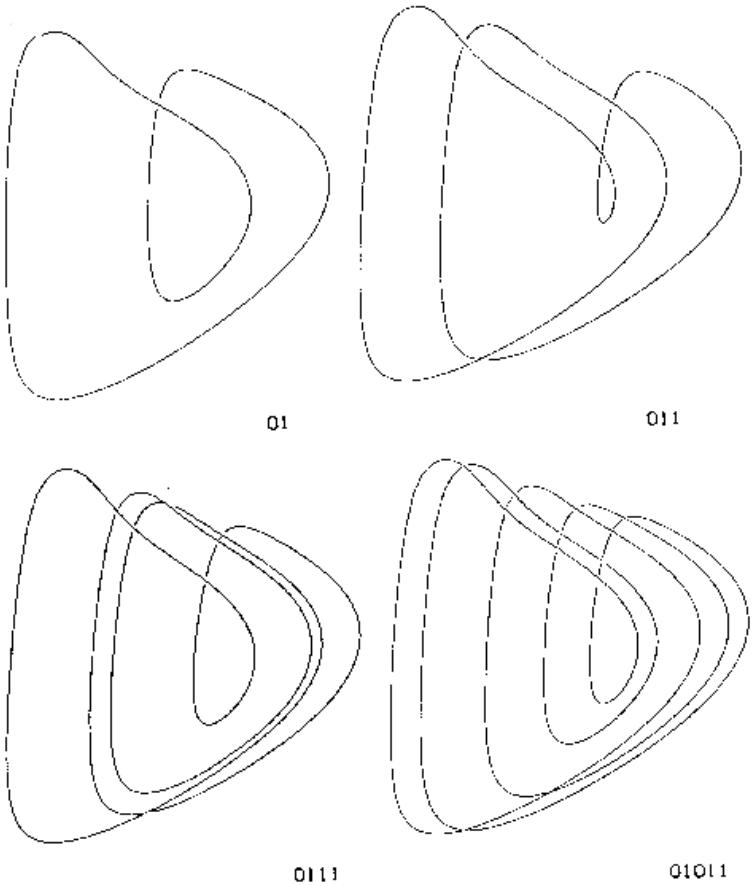
Once you have an embedding:

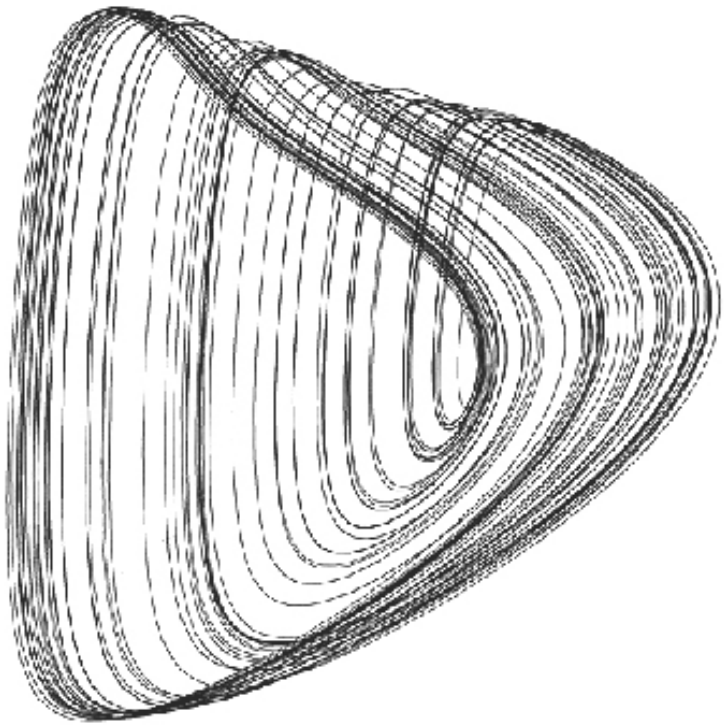
- Find a Poincaré Section
- Construct a First Return Map on the Section
- Introduce a Symbolic Encoding
- Encode all Unstable Periodic Orbits
- Find their Linking Numbers

Two Symbols Suffice! 0 and 1



Some Named Low-Period Orbits





Some Extracted and Reconstructed Periodic Orbits

Orbit	Name	Symbolics	Local Torsion	Self-Linking
1	1_1	1	1	0
2	2_1	01	1	1
3	3_1	011	2	2
4	4_1	0111	3	5
5	5_1	01 011	3	8
6	6_2	011 0M1	3	9
7	7_2	$(01)^2011$	4	16
8a	8_1	$(01)^20111$	5	23
8b	8_3	$01(011)^2$	5	21
9	9_3	$(01)^3011$	5	28
10a	10_6	$(011)^20101$	6	33
10b	10_6	$(011)^20111$	7	33
11	11_9	$01(011)^3$	7	40
13a		$(01)^2011\ 01\ 0111$	8	62
13b		$(01)^3011\ 0111$	8	60

Table of Experimental Linking Numbers

Orbit	Symbolics	1	2	3	4	5	6	7	8a	8b
1	1	0	1	1	2	2	2	3	4	3
2	01	1	1	2	3	4	4	5	6	6
3	011	1	2	2	4	5	6	7	8	8
4	0111	2	3	4	5	8	8	11	13	12
5	01 011	2	4	5	8	8	10	13	16	15
6	011 0M1	2	4	6	8	10	9	14	16	16
7	01 01 011	3	5	7	11	13	14	16	21	21
8a	01 01 0111	4	6	8	13	16	16	21	23	24
8b	01 011 011	3	6	8	12	15	16	21	24	21

[a] All indices are negative.

Testing the Result

(a), (c) y_1^m compared with y_1^d. (b), (d) y_3^m compared with y_3^d.