Chapter 1

Linear Systems

1.1 Linear Map of the Interval

1. $x' = \lambda x$
   - $0 < \lambda < 1$
   - $1 < \lambda < \infty$
   - $\lambda = +1$
   - $\lambda = -1$
   - $-1 < \lambda < 0$
   - $-\infty < \lambda < -1$
2. Fixed points: stable and unstable
3. Stability: Dynamical and structural
4. $x' = f(x; \lambda)$
5. State space
6. Control parameters
7. Affine map: $x' = ax + b$

1.1.1 Cobweb Diagrams

1.1.2 Fixed Points

1.1.3 Lyapunov Exponent

1.1.4 State Variables - Control Parameters

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1.1.5 Dynamical Stability

1.1.6 Structural Stability

1.2 One Dimensional Linear Flows

$\frac{dx}{dt} = \lambda x$

- $0 < \lambda < \infty$
- $\lambda = 0$
- $-\infty < \lambda < 0$

1.2.1 Simple (Canonical) Forms

1.2.2 Fixed Points

1.2.3 Lyapunov Exponent

1.2.4 Dynamical Stability

1.2.5 Structural Stability

1.3 Linear Map of R^2

1.3.1 Description by Matrices

1.3.2 Lyapunov Exponents

1.3.3 Fibonacci Sequences

1.3.4 Partition of Control Parameter Space: Tr and Det

1.3.5 Other Two-Dimensional Spaces

$R^1 \times S^1$, $S^1 \times S^1$, S^2, $\cdots$, Klein bottles, real projective spaces, stuff with handles, tori.

1.4 Two Dimensional Linear Flows

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Stability plot in Tr, Det space.

1.4.1 Simple Canonical Forms

1.4.2 Lyapunov Exponent

1.4.3 Dynamical Stability

1.5 Algorithms

1.5.1 The First Algorithm

$x_{n+1} = \frac{1}{2}(x_n + \frac{3}{x_n}) \rightarrow \sqrt{3}$

Newton's method

Cube roots of +1: Basins of Attraction

1.6 Two Simplest Nonlinear Equations

1. $\frac{dx}{dt} = \lambda x(1-x)$
2. $x' = \lambda x(1-x)$

1.7 Basins of Attraction

For square root algorithm

For cube root algorithm

For

$$\left(\frac{1}{x} + \frac{1}{1-x} + \frac{1}{x-2}\right) dx = \lambda dt$$

For potentials

1.8 Catastrophe Theory

Morse potentials, degenerate critical points, magic transformations, families of catastrophes, A_1 (normal critical p[oint), A_2 (fold), A_3 cusp, A_4 butterfly, etc.

Saddle-node bifurcations, symmetry-breaking bifurcations, period-doubling bifurcations.

Germs and unfoldings.