Nonlinear Dynamics

PHYS 471, 571

Problem Set # 3
Distributed Jan. 20, 2015
Due January 29, 2015

Undergraduates: Problems 1a, 2a, 3a and 4a.
Graduates: Problems 1b, 2b, 3b and 4a,b,c.
All students: Solutions must contain enough words so that I can understand what you think you did, and you will be able to understand what you did in 12 months. No words = No credit!

1. Bifurcation Diagram: Construct a bifurcation diagram
a. for the logistic map $x' = \lambda x(1-x)$ for $0 < \lambda \leq 4$.
b. for the logistic map $y' = a - y^2$ for $-\frac{1}{4} < a \leq 2$.

2. Escape Clause - a: Set $\lambda = 4.1$ in the map $x' = \lambda x(1-x)$. Choose uniformly spaced initial conditions in the range $x \in (0,1)$, count the number of iterates it takes for an iterate to become negative. Bin this number. Plot the binned distribution.
- b. Do this problem for $y' = a - y^2$ and $a = 2.05$

3. Caustics - a: Choose uniformly spaced initial conditions in the range $x \in (0,1)$ for $x' = \lambda x(1-x)$. Plot $f^{(3)}(x;\lambda)$ for the map $1 < \lambda \leq 4$. Say something useful about the structure of this plot. (Words like *singularity* are welcome.) Class questions about what to calculate and how to plot are welcome.
- b. Do this problem for $y' = a - y^2$ for $1/2 \leq a \leq 2$

4. Orbit Order: Plot caustics for the fifth and sixth iterations of the logsitic map $y' = a - y^2$.
a. Predict the relative order in which the (three) period five windows and the five period-six windows appear in the bifurcation diagram.

b. Compare with the results of Problem #1.

c. Determine the control parameter values a at which the three period five orbits are superstable (Newton's method or divide and conquer are recommended).

5. Lyapunov Exponent: Construct and plot the Lyapunov exponent for the map $y' = a - y^2$. Estimate the Lyapunov exponent at $a = 2$. Say something useful about the (negative) spikey structure of this plot.

6. Henon Conservtive Map: The area-preserving map introduced by Henon is often used to model synchrontron dynamics:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y - x^2 \end{bmatrix} \tag{1}$$

Set $\frac{\alpha}{2\pi} = 0.2050$. Choose a bunch of initial conditions. For each initial condition, iterate until the transients die out, then plot the next 1000 iterates. Your figure should look something like what appears below.

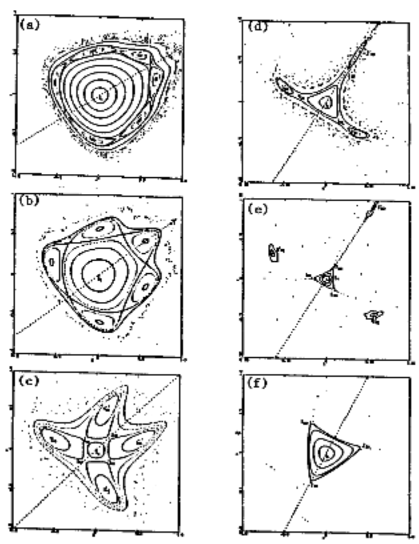

Figure 1: Some dynamics genrated by the quadratic Henon map for selected control parameter values. (a) $\alpha = 1.16$; (b) $\alpha = 1.33$; (c) $\alpha = 1.58$; (d) $\alpha = 2.00$; (e) $\alpha = 2.04$; (f) $\alpha = 2.21$.