QUANTUM MECHANICS III

PHYS 518

Problem Set # 4

Distributed: Oct. 25, 2013

Due: Nov. 1, 2013

1. Combining Transfer & Scattering Matrices: A scattering region I on the line can be described by a transfer matrix T and/or a scattering matrix S. The transmission amplitude is $t_1(E)$ and the reflection amplitude is $r_1(E)$.

a. Write down the transfer matrix in terms of the energy-dependent amplitudes $t_1(E), r_1(E)$.

b. Write down the scattering matrix in terms of the energy-dependent amplitudes $t_1(E), r_1(E)$.

c. A second scattering region II on the line is characterized by transmission and reflection amplitudes $t_2(E), r_2(E)$.

The two regions are separated by a distance L. Ditto **a, b** for this region.

d. Easy! Write down the transfer matrix for this combined potential.

e. HARD!! Write down the scattering matrix for this combined potential.

f. HHAARRDD but VERY INTERESTING: The scattering matrix constructed in **e** has the form: $S_{I\cup II} = M_1(M_2 - M_3)^{-1}M_4$ where you need to tell me what the 2×2 matrices M_i are. Expand this expression in a power series expansion (do not worry about convergence) and interpret the result in the language of "Feynman sum over all paths" formalism.

2. Transmission Resonance Peak: Assume the transmission amplitude for a scattering process is an analytic function with a single pole, viz: $t(E) = r_1/(E - z_1)$, where z_1 has positive real part and negative imaginary part: $z_1 = E_1 - i\Gamma/2$.

a. Sketch the time evolution of the corresponding scattering state (the wavefunction is *not* normalizable in a straightforward sense).

b. Plot the transmission probability $T(E) = |r_1/(E - z_1)|^2$. How must the parameters E_1, Γ_1, r_1 be related? Why?

c. What is the shape of the function $T(E)$ (somebody's name).

3. Resonance Interference: Now the transmission amplitude is the sum of (only) two poles:

$$t(E) = \frac{r_1}{E - z_1} + \frac{r_2}{E - z_2}$$

a. If the poles are "far apart" sketch what $T(E)$ looks like (ans: two Lorentzians). Give a condition for "far apart".

b. If the two resonances are not "far apart", then each "sees the other" and the interpretation as two independent Lorentzians must be renormalized. Write down the terms that lead to this problem.

c. Choose reasonable values that meet the condition of "not far apart" and plot $|t(E)|^2$. Tell me how you tweaked the parameters to get the plot that you show.

d. Describe the cross terms (in the absolute square) with language used to describe the two-slit interference experiment.