# QUANTUM MECHANICS III

# PHYS 518

## Problem Set # 1

## Distributed: Sept. 27, 2013

## Due: Oct. 4 (Pr. 1,2) and Oct. 7 (Pr. 3), 2013

**1. Avoided Crossings:** This Hamiltonian has been introduced to better understand 'avoided crossings' and the Landau-Zener effect:

$$H = \Delta E \frac{1}{2}\sigma_z \times \frac{t}{T_s} + p\sigma_x$$

Here $T_s$ sets the time scale for the transition through the crossing that can take place only if $p = 0$.

**a.** Find the eigenstates for arbitrary $t$.

**b.** Find the ground state for $t = -10T_s$

**c.** Compute and plot the inner product between the ground state at $t = -10T_s$ with the ground state for $t$ in the range $-10 \leq t/T_s \leq +10$.

**d.** What does this plot tell you? Put differently: how would you describe the physics embedded in this plot to an undergraduate who is learning Quantum Mechanics?

**2. Landau-Zener Amplitudes:** Landau (1932) and Zener (1932) showed that the *probability* for the transition $|1\rangle \rightarrow |1'\rangle$ is $e^{-2\pi\Gamma}$. $\Gamma = |p|^2/(\Delta E \times \hbar/T_s)$. The notation is as in Fig. 1 of Rubbmark et al., *Dynamical effects ...*, Phys. Rev. **A23**, 3107-3117 (1981). Explain why the following unitary matrix describes the transformation of the probability *amplitudes* through the avoided crossing:

$$\begin{bmatrix} |1'\rangle \\ |2'\rangle \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} |1\rangle \\ |2\rangle \end{bmatrix}$$

where $\cos\theta = e^{-\pi\Gamma}$.

**3. DUE OCT. 7 — Many Avoided Crossings:** Assume a system with many ($n$) levels undergoes many avoided crossings, as shown, for example, in Fig. 6 of Rubbmark et al., *Dynamical effects ...*, Phys. Rev. **A23**, 3107-3117 (1981). Model such a system by a Hamiltonian

$$H = \mathcal{E} + \mathcal{P} = \sum_i E_i(q)\Sigma_{ii} + \sum_{rs} p_{rs}\Sigma_{rs}$$

Here $\mathcal{E}$ is a diagonal matrix, $\mathcal{P}$ is hermitian, $\Sigma_{rs}$ is an $n \times n$ matrix, all of whose matrix elements are zero except for the matrix element in the $r$th row and $s$th column, which is $+1$; also $p_{rr} = 0$ and $p_{sr} = p_{rs}^*$.

Assume that there are only a finite number of crossings and that their "interaction boxes" are nonoverlapping. Describe (i.e., write an essay) how you would propagate the amplitudes of an arbitrary input wavefunction at "$t = -\infty$" to the output amplitudes at "$t = +\infty$" by multiplying a series of matrices "$M$" together. The matrix $M(ij)$ describes how the $i$th and $j$th levels cross, with

$$M(ij)_{kk} = 1 \quad (k \neq i, j), \quad M(ij)_{ii} = M(ij)_{jj} = \cos\theta_{ij}, \quad M(ij)_{ij} = ip_{ij}/|p_{ij}|\sin\theta_{ij}$$

What is $\cos\theta_{ij}$? (Don't get careless.)