QUANTUM MECHANICS III

PHYS 518

Problem Set # 5

Distributed: Nov. 15, 2010

Due: Nov. 22, 2010

1: GHZ States: Setup — Three particles are created in a localized region. They propagate away from each other with k-vectors $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ which define a plane. Each particle has two states. Measurements are eventually made on these three particles by devices that are spacelike separated from each other. Measurements are described by Pauli spin operators $\sigma^i_{x,y,z}$ for each particle. Choose the z direction for each particle in its direction of propagation, the x axis as the perpendicular direction out of the plane defined by the three k vectors, and choose the y direction to complete a local right-handed coordinate system. The operators for different particles commute: $[\sigma^i, \sigma^j] = 0$ for $i \neq j$. The operators for a single particle have the usual properties: $\sigma^i_x \sigma^i_y = i\sigma^i_z$, etc.

Define operators $X_1 = \sigma^1_x \sigma^2_y \sigma^3_y$, $X_2 = \sigma^1_y \sigma^2_x \sigma^3_y$, $X_3 = \sigma^1_y \sigma^2_y \sigma^3_x$ and $XXX = \sigma^1_x \sigma^2_x \sigma^3_x$. The operator X_1 represents a measurement of particle 1 in the x-direction and of particles 2 and 3 in the y direction, etc.

a. Show $X_i^2 = I_2^{\otimes 3} = I_8 = I$.

b. Show $XXX^2 = I$.

c. Compute $X_1 X_2 X_3$.

d. Show that these operators mutually commute.

e. Argue that each operator has two eigenvalues: ± 1.

f. Show that the three operators X_i can be used to label the eight states in the Hilbert space for these three particles.

g. Fill in the table:

State	Expression	X_1	X_2	X_3
$\|GHZ\rangle$	$(\|+++\rangle - \|---\rangle)/\sqrt{2}$	+	+	+
		−	−	−
		−	+	+
		+	−	−
		+	−	+
		−	+	−
		+	+	−
		−	−	+

(1)

h. Assume that the three particles are prepared in state $|GHZ\rangle$. Show that the product of measurements of σ_x for particle 1 and σ_y for particles 2 and 3 is +1. Similarly for measurements X_2 and X_3.

i. Use Einstein-like arguments to show that $m_x^1 m_y^2 m_y^3 = +1$, etc, and that this leads inexorably to Einstein-like predictions that the measurement of XXX leads to +1.

j. Use Bohr-like arguments to predict the measurement of XXX leads to −1.

k. What does the measurement give? Provide literature references.

Mermin-Peres Tic-Tac-Toe Operators:[1] With the notation as above, Mermin and Peres looked at the nine operators

$$\begin{array}{ccc} \sigma_x^1 & \sigma_x^2 & \sigma_x^1\sigma_x^2 \\ \sigma_y^2 & \sigma_y^1 & \sigma_y^1\sigma_y^2 \\ \sigma_x^1\sigma_y^2 & \sigma_y^1\sigma_x^2 & \sigma_z^1\sigma_z^2 \end{array} \tag{2}$$

a. Show that the three operators in each row commute.

b. Show that the three operators in each column commute.

d. Show that the product of the three operators in each row is +1.

e. Show that the product of the three operators in the first two columns is +1.

f. Show that the product of the three operators in the last column is −1.

g. Show that there is no way of assigning numerical values ±1 to each of the nine operators with these multiplicative properties.

h. Conclude with Peres that: Quantum theory is incompatible with the following propositions:

(1) The result of the measurement of an operator A depends solely on A and on the system being measured.

(2) If operators A and B commute, the result of a measurement of their product AB is the product of the results of separate measurements of A and of B.

i. Is this statement incompatible with Einstein's reality? With Bohr-Heisenberg Copenhagen dogma?

[1] A. Peres, Incompatible results of quantum measurements, Phys. Lett. A**151**, 107-108 (1990).)