The Hamiltonian describing an electron in a time-dependent external magnetic field is

\[ H = -\mu \cdot B = g \frac{e}{mc} \hbar (B_0 \sigma_z + B_1 \cos(\omega t)\sigma_x) = \frac{1}{2} \hbar \omega_0 \sigma_z + \frac{1}{2} \hbar \omega_1 \cos(\omega t)\sigma_x \]  

(1)

Here \( g \) is the electron gyromagnetic ratio, \(-e\) is the electron charge, \( g \frac{e}{mc} B_0 = \omega_0 \), etc. for the computations below choose \( \omega_0 = 5 \) and \( \omega_1 = 1 \).

1. Assume the electron starts off (\( t = 0 \)) in the ground state \( |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \).

Integrate the equations of motion for the spin-up and spin-down amplitudes \( a \) and \( b \) (\( \begin{pmatrix} a \\ b \end{pmatrix} \)) for the first few periods. Search over \( \omega \) to find the “resonance frequency”. Plot \(|a|^2 + |b|^2\) and \(|a|^2 - |b|^2\) at the resonance frequency as a function of time during this interval. If \(|a|^2 + |b|^2\) does not remain 1 during this time, what did/should you do? (Cosmopeople, MDpeople, think symplectic integration.)

2. Instead of computing the time evolution of the state function, compute the time evolution of the density operator. First, write \( \rho = \frac{1}{2} (I_2 + \sigma \cdot a) \) where \( a = \langle \sigma \rangle \). Then find the equation of motion for \( a \). It is a classical-like equation. Estimate the resonance frequency from this equation. Integrate this equation numerically for several periods at the estimated resonance frequency. Plot the analogs of \(|a|^2 + |b|^2\) and \(|a|^2 - |b|^2\). Also plot \( \langle \sigma_x \rangle \) during this interval and compare the result with \(|\text{trig. function}|^2\), where an obvious trig function is guessed.