QUANTUM MECHANICS III

PHYS 518

Problem Set # 3

Distributed: Oct. 8, 2010

Due: Oct. 15, 2010

1. An electron ($s = 1/2$) lives in the p level. It is subject to the Hamiltonian

$$\mathcal{H}(\theta) = 2\cos(\theta)\mathbf{L}\cdot\mathbf{S} + \mu\sin(\theta)\mathbf{B}\cdot(\mathbf{L}+\mathbf{2S}) \tag{1}$$

Perform your calculations in a convenient basis, for example: $\left| \begin{matrix} l & s \\ m_l & m_s \end{matrix} \right\rangle$.

a. Write down the 6×6 spin orbit coupling matrix for $\theta = 0$. What are its eigenvalues? eigenvectors?

b. Write down the Zeeman matrix for $\theta = \pi/2$. Assume $|\mu\mathbf{B}| = 1$. What are its eigenvalues? eigenvectors?

c. Compute and plot the eigenvalues of $\mathcal{H}(\theta)$ for $0 \le \theta \le \pi/2$. Make suitable observations about the eigenvalues and their behavior.

d. For any one interesting intermediate value of θ write down the 6 eigenvectors and make any remarks that are suitable.

e. For θ near 0 how do the energy levels depend on the external $\mathbf{B}$ field? (Hint: Landé g factor information is requested.)

f. For θ near $\pi/2$ how do the energy levels depend on the external $\mathbf{B}$ field?

2. The authors of *The Size of the Proton* probed the $2s_{1/2}F = 1$ to $2p_{3/2}F = 2$ transition in the $p^+\mu^-$ atom because ".. it has the largest transiton probability .." of all the transitions. Verify their claim.

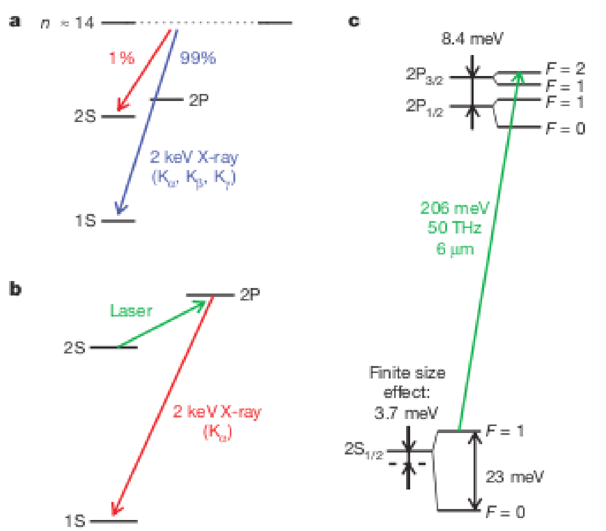

Figure 1 | Energy levels, cascade and experimental principle in muonic hydrogen. **a**, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The μp(2S) atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed K_α X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in μp. The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at $\lambda = 6\,\mu$m.