

QUANTUM MECHANICS III

PHYS 518

Problem Set # 2

Distributed: October 8, 2008

Due: October 17, 2008

In resonance with the Nobel Prize in Physics for 2008 on Symmetry Breaking^{1,2}: The *order parameter* describing the normal \rightarrow superconducting phase transition is the band gap Δ . Start from the BCS wavefunction (18.68) and the Hamiltonian (18.81):

- a. Compute $\langle BCS|H|BCS\rangle$.
- b. Compute $\partial\langle H - \mu N\rangle/\partial v_\alpha$.
- c. Determine the values of u_α, v_α that make $\langle H - \mu N\rangle$ stationary.
- d. Determine the ground state energy E_g (18.91).
- e. Derive the equation that determines the band gap Δ .
- f. Assume $\langle\alpha, -\alpha|V|\gamma, -\gamma\rangle = -V_0$ for $|\epsilon_\alpha - \mu| < \hbar\omega$ and assume that this matrix element is zero for $|\epsilon_\alpha - \mu| > \hbar\omega$. Determine Δ in terms of V_0 and the density of states at the Fermi surface.
- g. Under what conditions can the order parameter be nonzero?

1. Y. Nambu, Quasi-Particles and Gauge Invariance in the Theory of Superconductivity, Phys. Rev. **117**(3), 648-663 (1960).
2. Y. Nambu, Axial Vector Current Conservation in Weak Interactions, Phys. Rev. Lett. **4**(7), 380-382 (1960).