QUANTUM MECHANICS II

PHYS 517

Problem Set # 1

Distributed: April 3, 2015

Due: April 10, 2015

1. Angular Momentum Basics: Find the probability distributions of the orbital angular momentum operators L^2 and L_z for the following orbital state functions:
(a) $\psi(x) = R(r)\sin\theta\sin\phi$;
(b) $\psi(x) = R(r)\cos^2\theta$;
(c) $\psi(x) = R(r)\sin\theta\cos\theta\sin\phi$.

2. Angular Momentum J=1: Compute the angular momentum matrices J_x, J_y, J_z for $J = 1$.
(a) Construct the squares of each of these matrices.
(b) Show that these three squares commute.
(c) Construct their common eigenvectors.
(d) What is the sum of the three squares?

3. Probabilities: Particle 1 in a two-particle system has spin s_1 and particle 2 has spin s_2.
(a) What is the probability for the total spin to be S if particle 1 is in state $\left| \begin{array}{c} s_1 \\ m_1 \end{array} \right\rangle$ and particle 2 is in state $\left| \begin{array}{c} s_2 \\ m_2 \end{array} \right\rangle$?
(b) What is the probability for the total spin to be S if both of the particles are unpolarized?

4. Useful Identity: Prove

$$(\boldsymbol{\sigma} \cdot \mathbf{A})(\boldsymbol{\sigma}\cdot\mathbf{B}) = (\mathbf{A} \cdot \mathbf{B})I_2 + i\boldsymbol{\sigma}\cdot(\mathbf{A} \times \mathbf{B})$$

Here $\boldsymbol{\sigma}$ are the Pauli spin matrices and $\mathbf{A}$ and $\mathbf{B}$ are two operators that commute.

5. Spin-Dependent Potential: Two spin $\frac{1}{2}$ particles interact through a spin-dependent potential $V(r) = V_1(r) + \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} V_2(r)$. Show that the equation determining the bound states can be split into two equations, one having effective potential $V_1(r)+V_2(r)$ and the other having effective potential $V_1(r) - 3V_2(r)$.

6. Selection Rule: Show that a nucleus having spin 0 or spin $\frac{1}{2}$ cannot have an electric quadrulpole moment.

7. Quadrulpole Interaction: An electric quadrupole moment couples to the gradient of the electric field, or equivalently to the second derivative of the scalar electric potential Φ, with an interaction of the form $H_p = CS_iS_j\Phi_{ij}$.

(a) Show that this simplifies if one transforms to a principal axis coordinate system:

$$H_p = C\left\{S_x^2\Phi_{xx} + S_y^2\Phi_{yy} + S_z^2\Phi_{zz}\right\}$$

(b) Show that this hamiltonian can be further simplified to the form

$$H_p = A(3S_z^2 - \mathbf{S}\cdot\mathbf{S}) + B(S_+^2 + S_-^2)$$

How are A and B related to C?

(c) Find the eigenvalues of H_p for a system with $S = \frac{3}{2}$.