QUANTUM MECHANICS II

PHYS 517

Problem Set #5a

Distributed May 18, 2011

"Due" May 27, 2011

Thinking the Unthinkable:

Using random Number Generators to do Statistics

You are working in a Quantum Optics Laboratory as a computer jockey (they won't let you near a real $$$ laser for fear that my karma has rubbed off on you). Your job is to simulate the outcome of a series of measurements of the frequencies of photons emitted in an atomic transition.

1. Write down the spectral function $L(\nu)$ for a transition with center ν_0 and full-width at half height γ (a Lorentzian).

2. Construct the CDF* $C(\nu)$ from this PDF** by integration: $C(\nu) = \int_0^\nu L(\nu')d\nu'$. You may assume $\gamma \ll \nu_0$ so that the integral can be extended backward from 0 to $-\infty$.

3. Normalize this CDF so its range extends from 0 (at $-\infty$) to +1 (at $+\infty$).

4. Dust off your favorite random number generator and drive it around a couple of times until you are convinced that the wheels aren't about to fall off. Make it output random numbers in the interval $(0, 1)$.

5. Model data generated by 1000 measurements on this optical system. Do this as follows.

a. Output a random number r_1.
b. Set $C(\nu) = r_1$ and determine $\nu \to \nu_1$.
c. Save ν_1.
d. Do this again and again (999 more times).
e. Bin your results.
f. Compare with the original PDF $L(\nu)$.
g. How many events are outside the central range by more than 10γ?
h. How many are predicted to be outside the central range by more than 10γ?

* Cumulative Distribution Function ** Probability Distribution Function