QUANTUM MECHANICS II

PHYS 517

Problem Set #3

Distributed April 20, 2011

Due April 29, 2011

Density Matrices, Schwinger Representation

1. Ballentine, Problem 2.5, pp. 60-61.

2. Ballentine, Problem 2.6, p. 61.

3. Ballentine, Problem 2.8, pp. 61.

4. Ballentine, Problem 2.9, p. 62.

5. Ballentine, Problem 2.10, pp. 62.

6. Introduce creation $(a_1^\dagger, a_2^\dagger)$ and annihilation (a_1, a_2) operators for two independent boson modes, with standard commutation relations: $\left[a_i, a_j^\dagger\right] = \delta_{ij}$, all other commutators equal to zero. Now introduce three operators $\mathcal{S}_i = \mathbf{a}^\dagger(\frac{1}{2})\sigma_i\mathbf{a}$, where, for example, $\mathcal{S}_3 = \frac{1}{2}(a_1^\dagger a_1 - a_2^\dagger a_2)$.

a. Show that the operators $\mathcal{S}_i$ are hermitian.

b. Compute the commutators $[\mathcal{S}_i, \mathcal{S}_j]$.

c. Compute $\mathcal{S}^2 = \sum_{i=1}^3 \mathcal{S}_i^2$.

d. Show that $\mathcal{S}^2$ commutes with each $\mathcal{S}_i$.

e. Compute the matrix elements of the operators $\mathcal{S}^2$ in the direct product basis: $\langle n_1', n_2'| * |n_1, n_2\rangle$.

f. Compute the matrix elements of the operators $\mathcal{S}_i$ in the direct product basis: $\langle n_1', n_2'| * |n_1, n_2\rangle$.