QUANTUM MECHANICS II

PHYS 517

Problem Set #2

Distributed April 6, 2011

Due April 15, 2011

1. Rabi Oscillations: A two-level system is subject to a time-dependent hamiltonian

$$\mathcal{H} = \frac{1}{2}\begin{bmatrix} \hbar\omega_0 & \Omega e^{-i\omega t} \\ \Omega^* e^{+i\omega t} & -\hbar\omega_0 \end{bmatrix} \tag{1}$$

a. Give a physical interpretation for the three parameters that appear in this expression: $\hbar\omega_0, \Omega, \omega$.

b. Write down the equation satisfied by a suitable unitary transformation that propagates the state at time $t = 0$ to a state at any future time t.

c. Assume that the system starts in the ground state at time $t = 0$. Compute the probability that (a) the excited state is occupied at time t; (b) ground state is occupied at time t.

d. Plot these probabilities as a function of time for about two complete oscillations.

e. What function of these three variables determines the cycle time for this system?

2. Density Operator for 2-Level Systems:

a. Express the hamiltonian above in terms of the Pauli spin matrices

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

b. Write down a density operator for the 2-level system in terms of unknown parameters $a_\mu(t)$ and the Pauli spin matrices σ_μ ($\sigma_0 = I_2$).

c. Write down the dynamical equations of motion for the coefficients $a_\mu(t)$.

d. Integrate these equations and plot the three functions $a_i(t)$ for two cycles.

e. Express the expectation values $\langle\sigma_\mu\rangle$ in terms of the time-dependent functions $a_\mu(t)$.

f. Interpret your results assuming that the two states arise from a spin $\frac{1}{2}$ particle placed in a magnetic field.

3. Three-State Oscillations:
A hamiltonian describing the interaction among three states of a system is

$$\mathcal{H} = \begin{bmatrix} 99 & 7 & 5e^{+i\pi/4} \\ 7 & 21 & 3 \\ 5e^{-i\pi/4} & 3 & 1 \end{bmatrix}$$

Assume that at time $t = 0$ the system is in the 'ground state' $[0, 0, 1]^t$.

a. Compute and plot the probability that states $[1, 0, 0]^t$, $[0, 1, 0]^t$ and $[0, 0, 1]^t$ are occupied at later time t. Carry out the plots for about two cycles.

b. Identify the parameters in the hamiltonian responsible for the different periodicities.

$$\text{Jargon : bare states or mass states : } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Dressed states or 'flavor' states: eigenstates of the hamiltonian.