QUANTUM MECHANICS II

PHYS 517

Template for Midterm Exam

Hydrogen Atom

a. Draw an energy level diagram for the bound states of the nonrelativistic hydrogen atom. Show the energy scale (in eV), organize the energies in terms of the angular momentum values of the various levels, and identify the degeneracy of these levels, neglecting spin.

b. Show in this diagram how shielding by the inner electrons splits the N^2 fold degeneracy of the multiplet with quantum number N.

c. Use the 'Aufbau Principle' to derive the ground state configuration of ${}^{27}_{13}\mathrm{Al}$.

Nuclear Physics

Use the level filling scheme based on the energy level structure of (a) a three-dimensional harmonic oscillator, (b) perturbed by a spherically symmetric perturbation giving rise to an energy level spectrum of the form $-\gamma L^2$ ($\gamma > 0$), (c) followed by strong spin-orbit coupling of the form $-\delta \mathbf{l} \cdot \mathbf{s}$ that lowers the higher-j state compared with the lower-j state (Goeppert-Mayer's nuclear filling scheme), to construct the ground state configurations and identify the ground state spin, of the following nuclei:

a. Write down the nuclear ground state configuration of: ${}^{207}_{82}\mathrm{Pb}$, ${}^{208}_{82}\mathrm{Pb}$, and ${}^{209}_{82}\mathrm{Pb}$. For each, what the the ground state nuclear spin?

b. Write down the ground state configuration of all calcium isotopes from ${}^{40}_{20}\mathrm{Ca}$ to ${}^{48}_{20}\mathrm{Ca}$. For each, what is the ground state nuclear spin?

c. Write down the ground state configuration of all isoto*n*es from ${}^{48}_{20}\mathrm{Ca}$ to ${}^{56}_{28}\mathrm{Ni}$. For each, what is the ground state nuclear spin?

Crystal Field Splitting

An odd-even nucleus with an odd valence proton in a $j = \frac{7}{2}$ shell finds itself in a crystal where the crystal field produces an electrostatic perturbation with a dipole and a quadrupole contribution. The crystal field hamiltonian has the form $\mathcal{H} = DJ_z + Q(J_+^2 + J_-^2)$, where $D = \cos\theta$ and $Q = \sin\theta$.

a. Draw a diagram illustrating how the ground state energies split when $\theta = 0$.

c. Draw a diagram illustrating how the ground state energies split when $\theta = \frac{\pi}{2}$.

d. Draw a diagram illustrating how the ground state energies split as θ varies smoothly from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

Density Operators

A certain dynamical operator is represented by the 3×3 matrix $M = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$.

a. What is the spectrum of possible observed values when a measurement is made using this operator?

b. Assume that the state of the physical system is defined by the density operator $\rho = \begin{bmatrix} 1/2 & 0 & 1/4 \\ 0 & 1/4 & 0 \\ 1/4 & 0 & 1/4 \end{bmatrix}$. What is the expected value of the operator when a large number of observations has been made?

c. Assume a partcular measurement gives zero. What is the state density operator immediately following this measurement?

Time Evolution

Assume that the Hamiltonian that describes a 3-level system is a real symmetric matrix with nonzero offdiagonal matrix elements. Assume the basis states for the system are $|1\rangle, |2\rangle, |3\rangle$. Assume that at time $t = 0$ the system is in state $|1\rangle$. Describe *in detail* the steps you would take (have taken) in order to compute and plot the probability that the state $|i\rangle$ is occupied at time t.

Angular Momentum Coupling

a. Write down the wavefunction for a proton in a $d_{3/2}, m_j = \frac{1}{2}$ state in terms of its orbital (m_l) and spin (m_s) quantum numbers.

b. Write down the ground state wavefunction for a pair of valence neutrons in a $d_{3/2}$ orbital.