QUANTUM MECHANICS II

PHYS 517

Midterm Examination

May 11, 2011, 11:00 - 12:00

Instructions: Answer each question correctly in the bluebook.

Boundary conditions: 1 hour, open book (Ballentine); open notes, open homework; no collaboration; no graduate student assistants.

1. Hydrogen Atom: 40 pts.

a. Draw an energy level diagram for the bound states of the nonrelativistic hydrogen atom. Show the energy scale (in eV), organize the energies in terms of the angular momentum values of the various levels, and identify the degeneracy of these levels, neglecting spin.

b. Show in this diagram how shielding by the inner electrons splits the N^2 fold degeneracy of the multiplet with quantum number N.

c. Write down the ground state electronic configuration of $^{131}_{54}\mathrm{Xe}$.

d. For this electronic ground state what is: L, S, J?

e. What is the proton ground state configuration? What is the angular momentum of this state?

f. What is the neutron ground state configuration? What is the angular momentum of this state?

g. For the nuclear ground state, what is the spin I?

h. What is the total atomic spin $F = J + I$?

2. Crystal Field Splitting: 30 pts.

An odd-even nucleus with an odd valence proton in a $j = \frac{5}{2}$ shell finds itself in a crystal where the crystal field produces an electrostatic perturbation with a dipole and a quadrupole contribution. The crystal field hamiltonian has the form $\mathcal{H} = DJ_z + Q(J_+^2 + J_-^2)$, where $D = \cos\theta$ and $Q = \sin\theta$.

a. Draw a diagram illustrating how the ground state energies split when $\theta = 0$.

c. Draw a diagram illustrating how the ground state energies split when $\theta = \frac{\pi}{2}$.

d. Draw a diagram illustrating how the ground state energies split as θ varies smoothly from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

3. Density Operators 40 pts.

A certain dynamical operator is represented by the 3×3 hermitian matrix $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

a. What is the spectrum of possible observed values when a measurement is made using this operator?

b. Assume that the state of the physical system is defined by the density operator $\rho = \begin{bmatrix} 1/2 & 0 & 1/4 \\ 0 & 1/4 & 0 \\ 1/4 & 0 & 1/4 \end{bmatrix}$. What is the expected value of the operator when a large number of observations has been made?

c. Assume a partcular measurement gives $+1$. What is the state density operator immediately following this measurement?

4. Time Evolution 30 pts.

Assume that the Hamiltonian that describes a 4-level system is a real symmetric matrix with nonzero offdiagonal matrix elements. Assume the basis states for the system are $|1\rangle, |2\rangle, |3\rangle, |4\rangle$. Assume that at time $t = 0$ the system is in state $|1\rangle$. Describe *in detail* the steps you would take (have taken) in order to compute and plot the probability that the state $|i\rangle$ is occupied at time t.

5. Angular Momentum Coupling 30 pts.

a. Write down the wavefunction for a proton in a $p_{3/2}, m_j = \frac{-1}{2}$ state in terms of its orbital (m_l) and spin (m_s) quantum numbers.

b. Write down the ground state wavefunction for a pair of valence neutrons in a $d_{5/2}$ orbital.

6. Excitons 30 pts.

An exciton forms when a photon is absorbed by a semiconductor. This excites an electron from the valence band into the conduction band. In turn, this leaves behind a localized positively-charged hole. The electron in the conduction band is then attracted to this localized hole by the Coulomb force. This attraction provides a stabilizing energy balance. Consequently, the exciton has slightly less energy than the unbound electron and hole. The wavefunction of the bound state is said to be hydrogenic, an exotic atom state akin to that of a hydrogen atom. However, the binding energy is much smaller and the particle's size much larger than a hydrogen atom. This is because of both the screening of the Coulomb force by other electrons in the semiconductor (i.e., its dielectric constant), and the small effective masses of the excited electron and hole. The recombination of the electron and hole, i.e. the decay of the exciton, is limited by resonance stabilization due to the overlap of the electron and hole wave functions, resulting in an extended lifetime for the exciton. (This summary is from Wiki: Excitons.)

a. Estimate the energy emitted in a $2p \to 1s$ transition in a material in which the *effective* electron (and hole) mass is $m^* = 0.05 m_e$ and the dielectric constant is $\epsilon = 10$.

b. Estimate the size of the exciton in its ground state.