QUANTUM MECHANICS II

PHYS 517

Problem Set # 3
Distributed: Apr. 28, 2010
Due: May 5, 2010

1. Construct an analytic expression for the matrix element $t_{11}(E)$ of the transfer matrix for an electron in a simple binding potential of width $L$ (in Angstroms) and depth $V$ (in electron volts). Use asymptotic conditions $V_L = V_R = 0$. Choose $L$ in the range $4 \leq L \leq 16$ and $V$ in the range $-8 \leq V \leq 0$. Plot $t_{11}(E)$ vs. $E$ from $E = V + \epsilon$ to $E = 0 - \epsilon$ to determine the values of the bound state energies.

2. Construct a code to compute the transfer matrix for an arbitrary series of potentials $V_i$ of width $\delta_i$ (eV. and Ang.). Test your code by reproducing your plot of Problem #1.

3. Apply your code to a piecewise constant potential with three pieces: $(V_i, \delta_i)$, $i = 1, 2, 3$. Use $V_1 = V_3 = V$, where $V$ is the potential that you used in Problems #1 and 2. Use $\delta_1 = \delta_3 = L$, where $L$ is the length that you used in Problems #1 and 2. Use $V_2 = 0$ and $\delta_2 = L/2$. Assume as usual $V_{\text{left}} = V_{\text{right}} = 0$. Plot $t_{11}(E)$ vs. $E$ in the same energy range used previously. What can you say about the bound state energies?

4. Apply your code to a piecewise constant potential with five pieces: $(V_i, \delta_i)$, $i = 1, \cdots, 5$. Use $V_{\text{odd}} = V$, where $V$ is the potential that you used in Problems #1 and 2. Use $\delta_{\text{odd}} = L$, where $L$ is the length that you used in Problems #1 and 2. Use $V_{\text{even}} = 0$ and $\delta_{\text{even}} = L/2$. Assume the usual boundary conditions. Plot $t_{11}(E)$ vs. $E$ as above. What do you see? Do the bound state energies split? Remarks? Guesses? Extrapolate your guesses to a similar piecewise constant binding potential with $2n + 1$ pieces.