QUANTUM MECHANICS I

PHYS 516

Problem Set # 3

Distributed: Jan. 22, 2016

Due: January 29, 2016

1. Thermal Expectation Value: A harmonic oscillator with energy spacing $\Delta E = \hbar\omega$ is in thermal equilibrium with a bath at temperature T. Compute the mean energy of the oscillator, not forgetting to include the zero point energy.

2. Linear Chain: In one dimension, n particles, each of mass m, are coupled to each other by springs of spring constant k. The two masses at the ends are coupled to brick walls with similar springs.

a. Draw picture.
b. Compute the energy dispersion relation for the n modes.
c. What is the mean thermal energy in each mode?
d. What is the mean thermal energy in all modes taken together?
e. Set $T = 0$. What is the zero-point energy?

3. Quantum Surprise: Continuing the problem above ...
f. Place your finger on the mass at the kth position. What is the zero point energy in the subchain with masses $1, 2, \cdots k-1$? What is the zero point energy in the subchain with masses $k+1, ...n$?
g. Remove your finger and place it on the mass at position $k+1$. What is the zero point energy in the two subchains now?
h. Assume that the equilibrium spacing of the masses is a. What is the force on your finger when it is placed on the kth mass? And which direction is it in?

4. Mathematical Tricks: Like all the special functions of Mathematical Physics, the Hermite polynomials satisfy Recursion Relations, Differential Relations, and have Generating Functions:

Recursion Relations :	$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$	**22.7**
Differential Relations :	$\frac{d}{dx}H_n(x) = 2nH_{n-1}(x)$	**22.8**
Generating Function :	$e^{2zx-z^2} = \sum \frac{1}{n!}H_n(x)z^n$	**22.9**

Use the connection between these classical polynomials and the harmonic oscillator wavefunctions

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} \; H_n(x) \; e^{-x^2/2}$$

to construct Recursion Relations, Differential Relations, and Generating Functions for the harmonic oscillator wavefunctions.

Boldface points to tables in Abramowitz and Stegun.

5. Modify the code you wrote for Problem # 2 in Problem Set # 2 to compute the energy eigenvalues of the bimodal potential $V(x) = \frac{1}{4}x^4 - \frac{5}{2}x^2$. Print the six lowest eigenvalues and plot the corresponding eigenvectors. Discuss the results.