QUANTUM MECHANICS I

PHYS 516

Midterm Exam, Feb. 12, 2016

1. Scaling: The energy E and Bohr radius a_B for a hydrogen atom in its ground state are $E = -me^4/2\hbar^2 = -13.6\ eV$ and $a_B = \hbar^2/me^2 = 0.529$ Å. Estimate the ground state energy and radius of positronium.

2. Linear Chain: In one dimension, n particles, each of mass m, are coupled to each other by springs of spring constant k. The end masses are connected to brick walls by springs with spring constant k.

a. Guess the nature of the normal modes.

b. Construct the dispersion relation $\omega(\phi)$, where ϕ is an appropriate mode index $\phi = i2\pi m/(n+1)$.

c. Quantize this normal mode problem.

3. More Harmonic Oscillators: Three harmonic oscillators have energy spacing $\hbar\omega_1 = \hbar\omega_2 = 400 Mev$ and $\hbar\omega_3 = 600 MeV$. These oscillators share three excitations ($n_1 + n_2 + n_3 = 3$). Draw an energy level diagram, clearly indicating the energies and the degeneracies.

4. Diatomic Molecules: An imaginary diatomic molecule has an energy level spectrum given by the analytical expression

$$E(n,l) = \frac{(n+\frac{1}{2})\hbar\omega}{1+\alpha(n+\frac{1}{2})} \times \frac{l(l+1)\hbar^2}{2I_0(1-\beta l(l+1))}$$

Here I_0 is the moment of inertia and α, β are dimensionless.

Write down the 22 component $D_{2,2}$ in the Dunham energy expansion $E(n,l) = \sum_{p,q} D_{p,q}(n+\frac{1}{2})^p\,[l(l+1)]^q$.