QUANTUM MECHANICS I - III

PHYS 516 - 518

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Course Schedule: (Winter Quarter) MWF 11:00 - 11:50, Disque 919

Objective: To provide the foundations for modern physics.

Course Requirements and Obligations

Course grading will be based on assigned homework problem sets and a midterm and final exam.

Texts

Two texts and one supplement will be used for this course. The first text has been chosen from among many admirable texts because it provides a more comprehensive treatment of quantum physics discovered since 1970 than other texts.

The second text will be used primarily during the second quarter of this course (PHYS517). It provides hands-on experience for solving binding and scattering problems in one dimension and potentials involving periodic potentials, again in one dimension.

The third text (optional) is strongly recommended for those who feel their undergratuate experience in this beautiful subject may be deficient in some way. It is out of print but a limited number of copies are often available through Amazon in the event our book store has sold out of their reprinted copies.

David H. McIntyre
Quantum Mechanics
NY: Pearson, 2012 ISBN-10: 0-321-76579-6

R. Gilmore
Elementary Quantum Mechanics in One Dimension
Baltimore, Johns Hopkins University Press, 2004 ISBN 0-8018-8015-7

R. H. Dicke and J. P. Wittke
Introduction to Quantum Mechanics
Reading, MA: Addison-Wesley, 1960 ISBN 0-?

If it becomes a hardship to acquire this fine text, you can get about the same information from another, later, fine text:
David J. Griffiths,
Introduction to Quantum Mechanics,
Addison Wesley, 2004 (2^{nd} Edition)

In additon, supporting material will be distributed in class and mounted on-line on the course web site: http://einstein.drexel.edu/ bob/Physics-516_13.html

Structure & Organization of Course

Quantum Mechanics has evolved in three stages so far:

1. The Old quantum Theory of Bohr: b. 1913 - R.I.P. 1926.
2. Modern Quantum Theory: b. 1925-6 - alive and well in 2012.
3. Spooky Entanglement: b. 1964 and growing rapidly.

Modern Quantum Theory has been given three formulations:
1. Matrix Mechanics, by Born, Heisenberg, and Jordan in 1925.
2. Wave Mechanics, by Schrödinger in 1926.
3. Path Integrals, by Feynman in 1949.

These three formulations are equivalent. We will begin with Schrödinger's.

Just as Einstein had his *Annus Mirabilis* in 1905 (Special Relativity, photoelectric effect, Brownian motion), Schrödinger had his *Annus Mirabilis.* The year was 1926. During this year he created Quantum Theory in its Wave Mechanics formulation. This enormously creative year saw the publication of four fundamental papers on the quantum theory: Quantization as an Eigenvalue Problem: I - IV. The first two papers were separated from the second two by a pair of important papers. In one he showed the equivalence between his formulation and the matrix formulation of Born, Heisenberg, and Jordan. In the second of this intermediate pair he created *coherent states* for the harmonic oscillator. These now form the basis for all modern treatments of Quantum Optics. After 1927 Schrödinger left the field of quantum mechanics and did not make any further contributions. He returned briefly to this territory in 1935 with the indirect encouragement of Einstein to leave a severe criticism of his creation, now remembered under the general rubric of the "Schrödinger's Cat Paradox" (= *decoherence* in our modern language. This word was used for the first time by Schrödinger in his 1935 papers.)

We will begin by covering the topics in the four Schrödinger papers.

Quantization as an Eigenvalue Problem. I

1. Variational formulation.
2. Standard formulation.
3. Hydrogen atom: Bound states.
4. Hydrogen atom: Scattering states.

Quantization as an Eigenvalue Problem. II

1. Harmonic oscillator.
2. Rotator with fixed axis (2D).
3. Rigid rotator with free axis (3D).
4. Diatomic molecule.
5. Two-dimensional oscillators.
6. Three-dimensional oscillators.
7. Coupled oscillations.
8. Coherent states. (After the first of his 2 intermediate papers.)

Quantization as an Eigenvalue Problem. III

1. Perturbation theory.
2. Stark effect.
3. Line strengths.

Quantization as an Eigenvalue Problem. IV

1. Time-dependent wave equation.
2. Perturbation theory (time-dependent).
3. Resonance phenomena.
4. Minimal electromagnetic coupling.

Ehrenfest Theorems:

1. Expectation values and density matrices/operators.
2. Newton's Equations.
3. Harmonic motion.
4. Orbital angular momentum and torque.
5. Angular momentum and precession.

6. Lorentz force.
7. Hamilton's Equations.
8. The Virial.
9. Quadrupoles.
10. Euler's Equations.
11. Runge Lenz vector and precession (S.R. & G.R.)

Matrix Mechanics
1. Born, Heisenberg, and Jordan.
2. Schrödinger's demonstration of equivalence.
3. Then and Now: the Swing of the Pendulum.
4. Matrix computations.
5. FEM

Feynman's Path Integrals
1. A particle goes along all possible paths.
2. The Action Integral.
3. Equivalence with Schrödinger's Equation (time-dependent).
4. 2-Slit interference pattern (Young diffration pattern).
5. Single-Slit interference pattern (Fraunhofer diffration pattern).
6. Diffraction gratings.
7. Interferometers: Matrix methods.
8. Resonators: Matrix methods.
9. Networks: S-matrices.
10. Networks: eigenstates.

Broad Historical Sweep
1. The light dialogue: From Newton to Einstein (?) and Beyond?
2. The gravity dialogue: From Newton to Einstein (?) and Beyond?
3. Problems with ∞: Planck $\hbar$; Bohr atom; Renormalization; Casimir.
4. The Phases of Quantum Theory: 1913, 1926, 1964.
5. 1913: Correspondence Principle.
6. 1926: Ehrenfest Theorems.
7. 1935: EPR and Schrödinger's Cat.
8. 1964: Bell's Theorem unlocks the flood.
9. 2000 $\rightarrow$ "At last, we're free from our classical manacles." ("The Quantum world is weirder that we could possibly have imagined.")

Uncertainties
1. Position and momentum: $\Delta x \Delta p \geq \hbar/2$.
2. Time and energy: $\Delta t \Delta E \geq \hbar/2$.
3. Angle and angular momentum: $\Delta\theta\Delta L_\theta \geq \hbar/2$.
4. Number and phase: $\Delta N \Delta\phi \geq 1$.
5. Amplitude and phase: $\Delta A \Delta\phi \geq \pi$.

6. Light Blitz Box: 2 ships passing in the night.
7. Squeezed states: trading uncertainties.
8. COBE and an absolute rest frame.
9. Nyquist Theorem.
10. Cramer-Rao Uncertainty Relations.
11. Uncertainty Relations of Statistical Mechanics: $\Delta U \Delta \frac{1}{T} \geq k$.

Symmetry
1. Solving equations.
2. Symmetry $\Rightarrow$ degeneracy.
3. Dynamical symmetry.
4. Classification of states.
5. Point Groups, Space Groups.
6. $SU(2)$ and rotations.
7. $SU(3)$ and particles.
8. $SU(5)$
9. Symmetry-breaking.

Gauge Theories
1. Measuring the gravitational field.
2. Measuring the phase of an electric field.
3. Global gauge transformations: $U(1)$.
4. Local gauge transformations: $U(1)$.
5. Yang-Mills, Nuclear Forces and Mesons: $SU(2)$.
6. Utiyama.
7. Groups and gauge theories: gauge bosons.
8. Renormalizable gauge theories.

Troublesome Infinities
1. The Ultraviolet Catastrophe: Planck and $\hbar$.
2. The Hydrogen Catastrophe: Bohr and the Old Quantum Theory.
3. Electron Self-Energy Catastrophe: Renormalization Group Theory.
4. Zero-Point Fluctuation Catastrophes: Casimir Effect.

Quantum Theory: Phase III
1. Phase I: The Old Quantum Theory.
2. Phase II: Quantum theory: 1925 $\rightarrow$ present.
3. Phase III: The Great Smokey Dragon.
4. EPR & Schrödinger: Entanglement and Decoherence.
5. von Neumann's "proof"
6. Bohm's Hidden Variables: A Counterexample.
7. Bell's theorem (1964).
8. The first three measurements.
9. Later Measurements (Aspect).

10. The Floodgates are Opened: GHZ and others.
11. Entanglement at a Distance: The Danube.
12. Measuring Decoherence.
13. Looking at Pilot Waves (Yves Couder).
14. Delayed choice Experiment.
15. Quantum Eraser.
16. Bounding the speed of Quantum Information: V_{QI}/c.

C^3: Quantum Cryptography, Computing, Communication
(to be supplied)

Nobel Prizes for Contributions to Quantum Theory

Year	Recipient(s)	Contribution
1914	von Laue	Structure of matter
1915	Bragg, Bragg	X-ray cyrstallography
1918	Planck	$\hbar$ and ultra-violet catastrophe
1921	Einstein	Photoelectric effect
1921	Soddy	Transmutation of elements (Chemistry)
1922	Bohr	1^{st} version of Quantum Mechanics
1922	Ashton	Mass spectrometry of isotopes (Chemistry)
1925	Franck, Hertz	Internal atomic structure
1927	A. H. Compton, Wilson	Photon-electron scattering
1929	de Broglie	Particle-Wave duality
1930	Raman	Raman spectroscopy
1932	Heisenberg	Quantum Mechanics
1933	Schrödinger, Dirac	Quantum Mechanics
1934	Urey	Deuterium (Chemistry)
1935	Chadwick	Neutrons
1936	Anderson	Antiparticles
1937	Davison, G. P. Thompson	Diffraction of particles
1939	Lawrence	Cyclotron
1943	Stern	Magnetic properties of nuclei
1944	Rabi	Radiofrequency resonance techniques
1944	Hahn	Nuclear fission (Chemistry)
1945	Pauli	Exclusion Principle
1949	Yukawa	Strong force
1950	Powell	π and μ mesons
1951	Cockroft and Walton	Linacs and transmutation
1951	McMillan, Seaborg	More heavy isotopes (Chemistry)
1952	Bloch	Magnetic resonance
1954	Born, Bothe	Interpretation of Quantum Mechanics, spectroscopy
1955	Kusch, Lamb	QED measurements: anomalous moment, Lamb shift
1956	Bardeen, Brittain, Shockley	Transistor
1957	Lee, Yang	Parity violation
1959	Chamberlain, Segré	Antiproton
1961	Hofstadter, Mössbauer	Probing nucleons, Mössbauer effect
1962	Landau	Superfluidity
1963	Wigner, Goeppert-Mayer, Jensen	Symmetry (esp. parity), nuclear physics
1964	Townes, Basov, Prokhorov	Maser
1965	Feynman, Schwinger, Tomonaga	QED
1966	Kastler	Manipulating atomic states
1967	Bethe	Why stars shine

1969	Gell-Mann	$SU(3)$
1972	Bardeen, Cooper, Schrieffer	Superconductivity
1973	Josephson, Giaever, Esaki	Superconductive tunneling, tunnel junctions
1975	Bohr, Mottelson, Rainwater	Nuclear physics, neutrinoes
1976	Richter, Ting	Charmonium: J/ψ
1977	van Vleck, Anderson, Mott	Magnetism
1978	Kapitza	Superfluidity
1979	Glashow, Salam, Weinberg	Electroweak theory
1980	Cronin, Fitch	CP violation
1982	Wilson	Renormalization theory
1983	Chandrasekhar, Fowler	Nuclear physics and stars
1983	Rubbia, van der Meer	W, Z
1985	Klitzing	Quantized Hall effect
1986	Ruska, Binnig and Rorher	Electron optics, tunneling microscopy
1987	Bednorz, Müller	High temperature superconductors
1989	Ramsey, Paul, Dehmelt	Manipulating matter
1990	Friedman, Kendall, Taylor	"Observed" quarks
1994	Shull, Brockhouse	Neutron intererometry
1995	(Cowan), Reines, Perl	Discovered neutrinoes
1996	Lee, Osheroff, Richardson	Superfluid phase diagram
1997	Chu, Cohen-Tannoudji, Phillips	Laser cooling
1998	Tsui, Störmer, Laughlin	Fractional quantized Hall effect
1998	Kohn	Density functional theory (Chemistry)
1999	t'Hooft, Veltman	Renormalizability of electroweak theory
2000	Cornell, Weiman, Ketterle	BEC
2000	Heeger, MacDiarmid, Shirakawa	Conductive polymers (Chemistry)
2003	Abrikosov, Ginzburg, Leggett	Superconductivity and superfluidity
2004	Gross, Politzer, Wilczek	Asymptotic freedom
2005	Glauber, Hall, Hänsch	Quantum theory of optical ocherence, precision spectrosco
2007	Fert, Grünberg	Giant magnetoresistance
2007	Ertl	Surface chemistry (Chemistry)
2008	Nambu, Kobayashi, Maskawa	Symmetry-breaking
2009	Kao, Boyle & Smith	Fiber optics, CCDs
2010	Geim & Novoselov	Graphene
2012	Haroche, Wineland	Manipulating single particles
2013	(Brout) Englert and Higgs	'Higgs' particle
2014	Akasaki, Amano and Nakamura	'Blue Ray'

Course Topics

- Schrödinger's Papers
  1. Quantization as an Eigenvalue Problem: Part I
  2. Quantization as an Eigenvalue Problem: Part II
  3. Quantization as an Eigenvalue Problem: Part III
  4. Quantization as an Eigenvalue Problem: Part IV
- Forms of Quantum Theory: Matrix Mechanics, Wave Mechanics, Path Integrals
- Separation of Variables:
  1. Klein-Gordan Equation
  2. Schrödinger Equation
- Frobenius's Method
- Eigenvalues and Eigenvectors
- Brief Remarks: Spherical Harmonics
- Time-Independent Perturbation Theory
- Applictions:
  1. Finite nuclear size
  2. Zeeman Effect
  3. Stark Effect
  4. Crossed Fields
- Harmonic Oscillator
  1. Analytic solution: Frobenius' Method
  2. Operator solution
  3. Discretization and Matrix Diagonalization
  4. Ginzburg-Landau Quartic Potential
- Coupled Oscillators
  1. Linear Molecules and Normal Modes
  2. One-Dimensional Solids

    (a) One atom/unit cell
    (b) Two atoms/unit cell
    (c) Three atoms/unit cell
3. Two-dimensional solids
4. Three-dimensional solids

- Electromagnetic Field
  1. Maxwell's Equations
  2. Vector and Scalar Potentials
  3. Normal Modes
  4. Independent Oscillators
  5. Quantization
- Time Dependence
- Time-dependent perturbation theory
- Representations:
  1. Schrödinger
  2. Interaction
  3. Heisenberg
- Applictions:
  1. Perturbed harmonic oscillator
  2. Fermi golden Rule
  3. Lorentzians
- Angular Momentum
  1. Analytic representation, angular variables: L
  2. Algebraic representation, $|l, m_l\rangle$
  3. $J \simeq a^\dagger a$
  4. Spin angular momentum: S
  5. Total angular momentum: J
  6. Spherical harmonics
  7. Clebsch-Gordan coefficients
- Angular Momentum Applications
  1. Shielded Coulomb Potential $\rightarrow$ Mendelyeev
  2. Harmonic + Square Well + Spin Orbit = Nuclear Shell Model
  3. Hydrogen $\rightarrow$ Positronium $\rightarrow$ Charmonium $\rightarrow$ Bottomonium