QUANTUM MECHANICS I

PHYS 516

Problem Set # 6

Distributed: March 3, 2015

Due: March 12 (17), 2015

1. 2-Level Oscillations: The hamiltonian describing a two-level system is $H = \frac{\epsilon}{2}\sigma_z + \gamma\sigma_x$. At $t = 0$ the initial state is $\psi(t = 0) = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Plot $P(\uparrow, t)$ and $P(\downarrow, t)$ for $t \geq 0$.

2. Neutrinos with 2 Flavors: Assume neutrinos come with two flavors. The flavor eigenstates are not energy eigenstates. The two types of states are related by a 2×2 unitary transformation:

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

Use $\sin\theta_{12} = 0.1$.

a. Assume that a decay produces a flavor-1 neutrino. Resolve this flavor state into energy states.

b. Propagate the energy states forward in time from $t = 0$ to arbitrary time T.

c. Assume a neutrino detector has been set up a distance $L = cT$ away from the neutrino source to detect neutrinos with flavor 1 (or 2). Compute the probability *amplitude* for detecting neutrinos of either type. Use $E_2 = 10E_1 > 0$ and T large enough so your plots show interesting things.

d. Compute the probability for detecting neutrinos with flavor 1. Flavor 2.

e. Do the probabilities sum to $+1$? (**Hint:** they better!)

3. Rabi Oscillations: The hamiltonian describing a two-level system is

$$H = \begin{pmatrix} \frac{\epsilon}{2} & \gamma\cos\omega t \\ \gamma\cos\omega t & -\frac{\epsilon}{2} \end{pmatrix}$$

At $t = 0$ the initial state is $\psi(t=0) = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Plot $P(\uparrow, t)$ and $P(\downarrow, t)$ for $t \geq 0$ and "interesting choices" of ϵ, γ, ω.

4. Neutrinos with 3 Flavors: Assume neutrinos come with three flavors. The flavor eigenstates are not energy eigenstates. The three types of states are related by a 3×3 unitary transformation (neutrino mixing matrix):

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \begin{bmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{bmatrix} \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

Use $\sin^2 2\theta_{12} = 0.861$; $\sin^2 2\theta_{23} = 0.97$; $\sin^2 2\theta_{12} = 0.092$ (N.B: $\sin\theta_{13} \rightarrow \sin\theta_{13}e^{-i\delta}$ and we have set $\delta = 0$ for this problem set. There is one additional phase matrix that can be sandwiched between the last 3×3 matrix above and the mass column vector. It is a phase matrix

$$\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which carries physical signifficance only if neutrinos are Majorana particles. You may ignore this matrix for the purposes of thes problem.

a. Assume that a decay produces a flavor-1 neutrino. Resolve this flavor state into energy states.

b. Propagate the energy states forward in time from $t = 0$ to arbitrary time T.

c. Assume a detector has been set up a distance $L = cT$ away to detect neutrinos with any flavor. Compute the probability *amplitude* for detecting neutrinos each flavor. Use $E_3 = 6E_2 > 0, E_2 = 4E_1$ and T large enough so your plots show interesting things.

d. Compute the probability for detecting neutrinos of each flavor.

e. Do the probabilities sum to +1? (**Hint:** they better!)