QUANTUM MECHANICS I

PHYS 516

Problem Set # 5

Distributed: Feb. 27, 2015

Due: March 6, 2015

1. Zeeman and Stark Effects: In this problem set you will compute the effect of strong uniform external electric (Stark) and magnetic (Zeeman) fields on a valence electron in a sodium atom. In particular you will compute the energy eigenvalues of the ground state multiplet. The basis states are those of the valence electron in the absence of these external fields. They have quantum numbers $|Nlm\rangle$ with $N = 3$ and $l = 0, 1, 2$, $-l \leq m \leq +l$. Arrange these basis states $|lm\rangle$ in the following order: $|00\rangle, |1+1\rangle, |10\rangle, |1-1\rangle, |2+2\rangle, |2+1\rangle, |20\rangle, |2-1\rangle, |2-2\rangle$.

a. Assume $E_{3p} - E_{3s} = 2$ and $E_{3d} - E_{3s} = 3$. Here E_{Nl} are the energy levels of the unperturbed states in the sodium atom. The energies have been scaled to dimensionless units so that numerical computations can be easily carried out. Write down the 9×9 matrix H_0 (it is diagonal).

b. An external magnetic field is applied. Its contribution to the perturbation hamiltonian is $-\mu \cdot \mathbf{B}$ where $\mu = e\hbar/2mc$ is the Bohr magneton. Write down the matrix elements of L_z. Show that the energy level spectrum of L_z is:

$$(-2)^1(-1)^2(0)^3(+1)^2(+2)^1 \tag{1}$$

c. Write down the matrix elements of L_x. Diagonalize this matrix and show that the energy level spectrum is given by Eq. (1). This is a sanity check on your computation of the matrix elements.

d. An external electric field is applied. Its contribution to the perturbation hamiltonian is $-\mathbf{d} \cdot \mathbf{E}$, where $\mathbf{d}$ is the dipole $-e\mathbf{x}$. Choose your coordinate axis in the direction of the external electric field and compute the matrix elements of $-e|\mathbf{E}|z$. Diagonalize this matrix and show that the energy level spectrum is proportional to that given in Eq. (1). What is the proportionality factor? Set this factor equal to 1 so that the electric and magnetic field perturbations have the same spectrum if applied separately.

e. The hamiltonian describing the entire system is

$$H = H_0 + H_{\text{pert}} = H_0 - e|\mathbf{E}|z - \mu|\mathbf{B}|\left(L_z \cos\theta + L_x \sin\theta\right)$$

Diagonalize this matrix for $0 \leq \theta \leq 2\pi$ and plot the nine eigenvalues. You should find something like what is shown in Fig. 1).

f. Say something intelligent about the kinks in the second and third excited states in this figure.

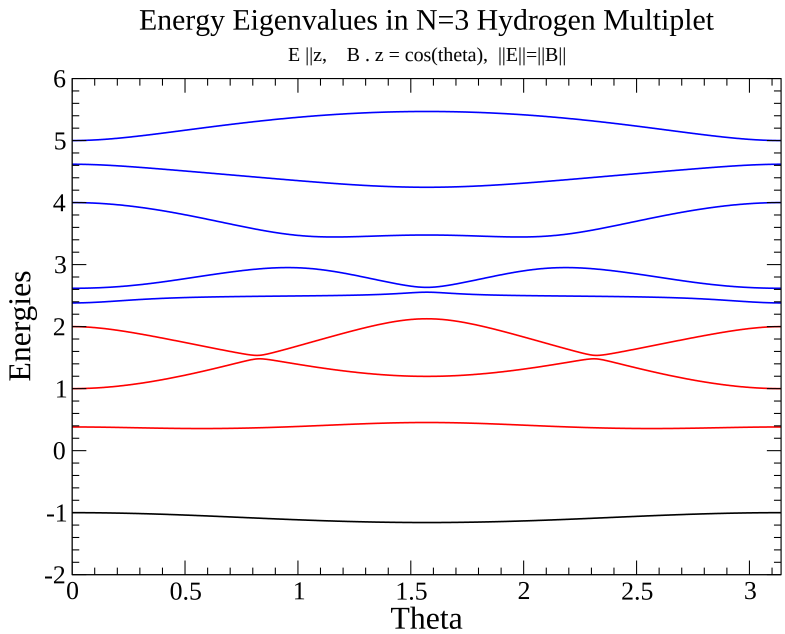

Figure 1: Energy levels of the ground state multiplet of an electron (no spin) in external electric (along z axis) and magnetic field, with $\mathbf{B} \cdot \mathbf{E}/|\mathbf{B}||\mathbf{E}| = \cos\theta$.