QUANTUM MECHANICS I

PHYS 516

Problem Set # 5
Distributed: Feb. 24, 2014
Due: February 29, 2014

1. 2-Level Oscillations: The hamiltonian describing a two-level system is $H = \frac{\epsilon}{2}\sigma_z + \gamma\sigma_x$. At $t = 0$ the initial state is $\psi(t = 0) = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Plot $P(\uparrow, t)$ and $P(\downarrow, t)$ for $t \geq 0$.

2. Rabi Oscillations: The hamiltonian describing a two-level system is

$$H = \begin{pmatrix} \frac{\epsilon}{2} & \gamma\cos\omega t \\ \gamma\cos\omega t & -\frac{\epsilon}{2} \end{pmatrix}$$

At $t = 0$ the initial state is $\psi(t = 0) = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Plot $P(\uparrow, t)$ and $P(\downarrow, t)$ for $t \geq 0$ and "interesting choices" of ϵ, γ, ω.

3. Spreading Wavefunctions: $\psi(x, t = 0) = Ne^{-x^2/2\sigma^2}$.
a. Compute $\psi(x, t > 0)$.
b. Plot $|\psi(x, t)|^2$ for several well-chosen values of t.
c. How long does it take for the wave function width to double?

4. Harmonic Oscillations: $\psi_n(x)$ are the normalized harmonic oscillator wavefunctions (assume $m = k = \hbar = 1$). Define $\Phi_n(x, t) = \psi_n(x)e^{-\frac{i}{\hbar}E_n t}$.
a. Plot $|\Phi_0(x, t)|^2$.
b. Plot $|\Phi_1(x, t)|^2$.
c. Plot $|\frac{1}{\sqrt{2}}(\Phi_0(x, t) + \Phi_1(x, t))|^2$.
d. Set $\bar{x}(t) = \langle\Phi(x, t)|x|\Phi(x, t)\rangle$, where $\Phi(x, t)$ is the wavefunction defined in **c.** Plot $\bar{x}(t)$.