QUANTUM MECHANICS I

PHYS 516

Problem Set # 1

Distributed: Jan. 8, 2014

Due: January 17, 2014

1. Scaling: For each of these pairs compute the binding energy and the size (diameter):

System	Energy	Size
hydrogen atom: p^+e^- (nonrelativistic)	13.6 eV	1.058 Å(diam.)
$\mathrm{He}^{\mathrm{II}}$:		
Cu^{29}: (nonrelativistic)		
Cu^{29}: (relativistic)		
mu-mesic atom: $p^+\mu^-$		
pi-mesic atom: $p^+\pi^-$		
positronium: e^+e^-		
muonium: $\mu^+\mu^-$		
pionium: $\pi^+\pi^-$		
Si exciton: $\epsilon = 11.9, m_e = 0.8m, m_h = 0.4m$		
GaAs exciton: $\epsilon = 12.5, m_e = 0.07m, m_h = 0.4m$		

El^{I} is neutral Element, and El^{n+1} is Element without n of its electrons. For excitons the electron (m_e) and hole (m_h) effective masses are given as multiples of the free electron mass. Recall that the mass m used in expressions for the hydrogen atom properties is the proton-electron reduced mass.

2. Relativistic Schrodinger Equation: Schrödonger solved the relativistic problem before he proposed his nonrelativistic equation. You will do that here

a. Write down the relativistic equation for a spinless electron in the presence of a spinless proton.

b. Use separation of variables to "get rid of" the angular dependence.

c. Use the useful transformation $R(r) = \frac{1}{r}f(r)$ and write down the radial equation in terms of the unknown function $f(r)$.

d. Compare this equation with an equation in Table 22.6 from Abramowitz and Stegen. What do you conclude?

e. Show

$$E(n, l, \alpha) = \frac{mc^2}{\sqrt{1 + \frac{\alpha^2}{\left(n + \frac{1}{2} + \sqrt{(l + \frac{1}{2})^2 - \alpha^2}\right)^2}}} \tag{1}$$

f. What effect does relativity have on the "Bohr radius" of an electron?