QUANTUM MECHANICS I

PHYS 516

Problem Set # 8

Distributed: March 1, 2013

Due: March 11, 2013

1. Density Matrices and Max Born Probabilities — Ballentine Problem 2.8: Compute $P(M = 0|\rho)$ for the operator M and the density matrices ρ_i:

$$M = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \rho_a = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & {}^{1}\!4 \end{bmatrix} \quad \rho_b = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \quad \rho_c = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

2. Spin Flips: A two-level system (e.g., spin) is described by a hamiltonian ($\mathbf{B}$ is in the x-y plane)

$$H = \frac{\epsilon}{2}\sigma_z + \sigma \cdot \mathbf{B}$$

a. Construct the unitary transformation that describes the time evolution of this quantum system.

b. Assume the system is in its ground state at time $t = 0$: $\psi(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Plot the occupation probability for the ground and excited states as a function of time for about three periods. Choose $\epsilon = 1$ and $|\mathbf{B}| = 1$.

c. Construct the density operator $\hat{\rho}(t)$ from $\hat{\rho}(t = 0)$.

d. Compute the expectation values of the Pauli spin operators σ_i as a function of time.

3. Neutrinos with 2 Flavors: Assume neutrinos come with two flavors. The flavor eigenstates are not energy eigenstates. The two types of states are related by a 2×2 unitary transformation:

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

Use $\sin\theta_{12} = 0.1$.

a. Assume that a decay produces a flavor-1 neutrino. Resolve this flavor state into energy states.

b. Propagate the energy states forward in time from $t = 0$ to arbitrary time T.

c. Assume an experiment has been set up a distance $L = cT$ away to detect neutrinos with flavor 1 (or 2). Compute the probability *amplitude* for detecting neutrinos of either type.

d. Compute the probability for detecting neutrinos with flavor 1. Flavor 2. Use $E_2 = 10E_1 > 0$ and T large enough so your plots show interesting things.

e. Do the probabilities sum to +1? (**Hint:** they better!)

4. Neutrinos with 3 Flavors: Assume neutrinos come with three flavors. The flavor eigenstates are not energy eigenstates. The three types of states are related by a 3×3 unitary transformation:

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \begin{bmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{bmatrix} \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

Use $\sin^2 2\theta_{12} = 0.861$; $\sin^2 2\theta_{23} = 0.97$; $\sin^2 2\theta_{12} = 0.092$ (N.B: $\sin\theta_{13} \to \sin\theta_{13} e^{-i\delta}$ and we have set $\delta = 0$ for this problem set.

a. Assume that a decay produces a flavor-1 neutrino. Resolve this flavor state into energy states.

b. Propagate the energy states forward in time from $t = 0$ to arbitrary time T.

c. Assume an experiment has been set up a distance $L = cT$ away to detect neutrinos with any flavor. Compute the probability *amplitude* for detecting neutrinos each flavor.

d. Compute the probability for detecting neutrinos of each flavor. Use $E_3 = 6E_2 > 0, E_2 = 4E_1$ and T large enough so your plots show interesting things.

e. Do the probabilities sum to +1? (**Hint:** they better!)