QUANTUM MECHANICS I

PHYS 516

Problem Set # 7

Distributed: Feb. 20, 2013

Due: Mar. 1, 2013

1. Electron Screening: An electron is dropped into the Coulomb potential supplied by a helium nucleus. It eventually winds up in its ground state.

a. What is its binding energy?

b. What can you say if you add up the energies of all the photons emitted during the deexcitation process:

c. What is the ground state wave function?

d. What is the probability distribution of the electron?

e. Plot $Z(r)$ vs. r, where $Z(r)$ is the effective charge seen by a "test particle" a distance r from the nucleus. Plot in terms of the (hydrogen atom) Bohr radius a_B.

2. Phonon Dispersion Relation: A one-dimensional lattice has N unit cells with three atoms/unit cell. The atoms have masses $m_1 = 2,\ m_2 = 5,\ m_3 = 3$ and the nearest neighbor interactions among them are described by spring constants $k_{3'1} = 1, k_{12} = 4, k_{23} = 3$. In addition, there is one next-nearest neighbor interaction between atoms 3 in cell $i-1$ and 2 in cell i with spring constant $k_{3'2} = 0.7$. Apply periodic boundary conditions to compute and plot the dispersion relation(s) for the three phonon bands supported by this lattice.

3. Scaling Again: Estimate the diameter of an exciton in GaAs. Use $\epsilon = 12.5$, $m_e = 0.07m_0$, and $m_h = 0.4m_0$, where m_e and m_h are the electron and hole effective masses in the semiconductor expressed in terms of the free-space electron mass m_0.

4. Problems in RelativityLand: Compute the energy of a single spinless relativistic electron in its ground state around a nucleus of charge Z when:

a. $Z = 68$.

b. $Z = 70$.

Use the Klein-Gordan expression for energies:

$$E(n, l, \alpha) = \frac{mc^2}{\sqrt{1 + \frac{\alpha^2}{\left(n + \frac{1}{2} + \sqrt{(l + \frac{1}{2})^2 - \alpha^2}\right)^2}}} \tag{1}$$

Remarks?