QUANTUM MECHANICS I

PHYS 516

Problem Set # 4

Distributed: Jan. 25, 2013

Due: Feb. 4, 2013

1. Adiabatic change: In an infinitely deep square well potential of length L the eigenstates of a particle of mass m are the well-known states $\phi_k(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{k\pi x}{L}\right)$. In an electrostatic potential $V_0 \sin\frac{\pi x}{L}$ in this well the eigenstates are $\psi_k(x)$. You have previously computed these for some V_0.

A particle of mass m is placed inside the infinitely deep one-dimensional square well potential of length L. $V_0 = 0$. The particle eventually gets into its ground state $\phi_1(x)$.

An electrostatic potential of the form $V_0 \sin\frac{\pi x}{L}$ is placed in the potential and V_0 is *slowly* ("adiabatically") increased from 0 to $V_0 = 16$.

a. What is the final state? Why? Plot this state.

b. What is the energy of this state?

c. How much work was done?

d. Explain why you think your calculations have converged. If you think they might not have converged, redo parts **b.**, **c.**, and **d.** until you are convinced they have.

2. Sudden changes: The particle is in the potential described above, with $V_0 = 16$, in its ground state $\psi_1(x)$. V_0 is suddenly changed from $V_0 = 16$ to 0.

a. An energy measurement is made. What are the possible outcomes $E_1, E_3, E_5, \cdots$ and what are the probabilities of these outcomes?

Assume the measurement shows that the particle is in its ground state $\phi_1(x)$. Now the potential is suddenly changed from $V_0 = 0$ to $V_0 = 16$.

b. An energy measurement is made again. What are the possible outcomes $E_1, E_3, E_5, \cdots$ and what are the probabilities of these outcomes?

3. Finite Differences: A particle of mass m is placed inside an infinitely deep one-dimensional square well potential of length L. Approximate the kinetic energy term by the standard finite-difference expression:

$$\frac{d^2\psi}{dx^2}_{|j} = \frac{\psi_{j+1} - 2\psi_j + \psi_{j-1}}{(\Delta x)^2}$$

The potential is $V(x) = 0$.

a. Compute the energy eigenvalues. As usual, set $m = \hbar = 1$ and $L = 10$ (and don't forget $\frac{1}{2} \to \frac{1}{2}!$). Plot the eigenvalues computed by the finite difference method (on the vertical axis) against those available analytically (on the horizontal axis). Remarks?

b. Plot the eigenvectors associated with the five lowest eigenvalues. On the same graph plot the analytically available eigenvectors. Why are the normalizations different?

c. What is the scale factor that needs to be introduced to bring these plots into agreement (up to sign)? Explain the scale difference between the analytic and numerical calculations.

4. Problem Set # 2 redux: Now set $V(x) = V_0 \sin \frac{\pi x}{L}$ with $V_0 = 16$. Compute the five lowest energy eigenvalues using the finite difference method of Problem #3 and compare with those of Problem #1. Plot the ground state as computed in this problem (properly scaled) and as computed in Problem #1 on the same graph. Comments?

5. Numerical Treatment of the Harmonic Oscillator: Repeat the computation of Problem #3 but with two changes: (a): $-\infty < x < +\infty$, (b) $V(x) = \frac{1}{2}kx^2$. As usual, $m = \hbar = k = 1$.

a. Compute the energy eigenvalues. Plot the eigenvalues computed by the finite difference method (vertical) with those available analytically (horizontal). Remarks?

b. Plot the eigenvectors associated with the five lowest eigenvalues. On the same graph plot the analytically available eigenvectors.

c. What gives you confidence (or lack) that your calculation has converged for these eigenvectors?