QUANTUM MECHANICS I

PHYS 516

Problem Set # 3
Distributed: Jan. 18, 2013
Due: Jan. 25, 2013

1. Solution of NR Schrödinger Equation for H atom: Apply the quantization condition for square integrable solutions of the equation

$$\left(\frac{d^2}{dr^2} + \frac{A}{r^2} + \frac{B}{r} + C \right) R(r) = 0$$

to the nonrelativistic equation for an electron in a Coulomb potential to determine the energy eigenvalue spectrum.

2. NR Limit of Klein-Gordan Energy: Expand the expression for the energy of a relativistic electron without spin (solution of the Klein-Gordan equation) in powers of the fine structure constant α up to and including order six. Recall

$$E(n, l, \alpha) = \frac{mc^2}{\sqrt{1 + \frac{\alpha^2}{\left(n + \frac{1}{2} + \sqrt{(l+\frac{1}{2})^2 - \alpha^2} \right)^2}}} \tag{1}$$

Compare the first correction to the rest energy with the nonrelativistic spectrum.

3. Scaling and Problems: Compute the energy of a single electron in its ground state around a nucleus of charge Z when:
  a. $Z = 68$.
  b. $Z = 70$.

4. More Scaling: What effect does relativity have on the "Bohr radius" of an electron?

5. Harmonic Oscillator:

a. Write down the Schrödinger equation for the harmonic oscillator, with $V(x) = \frac{1}{2}kx^2$.

b. Find the solution using Table 22.6 from Abramowitz and Stegun. What combination of parameters $\hbar, k, m$ gives the length scale?

c. What is the energy eigenvalue spectrum?

d. Write down the eigenfunctions.

e. Plot the eigenfunctions associated with the five lowest eigenvalues.

f. Plot $|\psi_{20}(x)|^2$ vs. x. What is this probability distribution function trying to tell you?