QUANTUM MECHANICS I

PHYS 516

Problem Set # 2

Distributed: Jan. 9, 2013

Due: Jan. 18, 2013

1. Schrödinger proposes that the wavefunction for a physical system is real and satisfies a variational condition

$$\delta \int \left(\frac{\hbar^2}{2m} (\nabla \psi)^2 + V(x)\psi^2 \right) dx = 0 \qquad\qquad \int \psi^2 dx = 1 \tag{1}$$

The normalization condition is to prevent the trivial solution $\psi = 0$ from occurring. He then carries out an integral by parts to convert this expression to a second-order partial differential equation.

Carry this step out to obtain the Schrödinger equation. **Explain** each of the steps that you take in this process. How do you treat the normalization condition and the boundary conditions (surface integral)? What is the Schrödinger equation for a Coulomb potential?

2. Solve the Schrödinger equation for a particle of mass m confined to a one-dimensional infinitely deep square well of length L. Write down the normalized (to +1) wavefunctions $\phi_j(x)$. Set $m = \hbar = 1$ and $L = 10$ and write down the energy level spectrum.

3. At the bottom of the first page of Schrödinger's first paper on Wave Mechanics is the sentence "... equation (1') can always be transformed so as to become a quadratic form (of ψ and its first derivatives) equated to zero."

Assume a potential of the form $V_0 \sin(\pi x/L)$ with $0 \leq x \leq L$ and $V(x) = \infty$ outside this range. Assume as above that $L = 10, m = 1, \hbar = 1$. Assume $\psi = \sum a_j \phi_j(x)$, where $\phi_j(x)$ are solutions for the square well constructed in Problem #2.

(a) Construct the quadratic form that Schrödinger speaks of for this problem. Use at least 10 basis functions. Use V_0 as assigned below.

(b) Find the minimum energy eigenvalue.

(c) Construct the ground state wavefunction.

(d) Plot the ground state wavefunction and its square.

Brewer	-12	Bridstrup	-10	Brown	-8
Goren	-6	Huynh	-4	Martin	-2
Plowman	2	Schlenker	4	Stone	6
Timlin	8	Wolfe	10	Young	12