QUANTUM MECHANICS I

PHYS 516

Problem Set # 4
Distributed: February 17, 2012
Due: February 27, 2012

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2 + V_0 e^{-x^2}$$

1. Diagonalization in Coordinate Basis: Find the 6 lowest eigenvalues and corresponding eigenvectors for the hamiltonian above by using a discretization of the second order differential operator. Set $m = k = 1$ and $V_0 = 6$. Plot the 6 lowest eigenvectors.

2. Diagonalization in Harmonic Oscillator Basis: Find the 6 lowest eigenvalues and corresponding eigenvectors for the hamiltonian above by computing the matrix elements of the term $V_0 e^{-x^2}$: $V_0\langle p|e^{-x^2}|q\rangle$ and adding this matrix to the diagonal matrix $\langle p|\frac{p^2}{2m} + \frac{1}{2}kx^2|q\rangle$. Plot the 6 lowest eigenvectors and compare your results with those obtained in Problem # 1.

3. Phonons — I: N particles of mass m are connected in a linear chain with $N-1$ identical springs with spring constant k. Each end mass is connected to a brick wall by a spring with the same spring constant.

a. Write down the Lagrangian.

b. Write down the Euler-Lagrange equations in matrix form.

c. Compute the eigenvalues of this set of equations (no need to compute the eigenvectors).

d. Write down the hamiltonian for the normal modes of this mass-spring chain. For each mode, you need to know either: the effective masses and spring constants; or the normal mode frequencies.

e. Quantize.

f. Compute the partition function $Z(T)$, where T is equilibrium temperature.

g. What is the zero-point energy of this linear chain?