QUANTUM MECHANICS I

PHYS 516

Problem Set # 3
Distributed: February 6, 2012
Due: February 13, 2012

1. Analytic Computation: The eigenfunctions of the one-dimensional harmonic oscillator are

$$\psi_n(x) = \frac{H_n(x)}{\sqrt{2^n n! \sqrt{\pi}}} \; e^{-x^2/2}$$

in dimensionless units.

a. Plot the lowest five eigenfunctions.

b. Plot $|\psi_{20}(x)|^2$. Explain what this probability distribution is trying to tell you.

2. Numerical Computation: Discretize an appropriate part of the real line to convert the Schrödinger wave equation to a matrix eigenvalue equation.

a. What interval did you discretize? Why? How many points in this interval? Why?

b. Describe (in words) the structure of the kinetic energy matrix.

c. Describe (in words) the structure of the potential energy operator.

d. Diagonalize the hamiltonian. Sort the output from smallest to largest eigenvalue.

e. Plot the eigen*values*. Compare them to the analytically available eigenvalues. Which eigenvalues/vectors would you trust?

f. Plot the eigenvectors with the five smallest eigenvalues.

g. Compare them with the plots you obtained in Problem #1. You can do this comparison by plotting the analytic and numerically obtained eigenfunctions in the same plot.

h. Do the plots agree? If not, why not? How do you reconcile this difference? If you found a difference, what is it due to and how do you fix it?

3. Numerically compute the five lowest eigenvectors for a particle in an infinitely deep square well potential well of length 4 units. What are the eigenvalues? How do they compare with those available from analytic calculations? Plot the analytic and numerical eigenvectors on the same graph. Comments?

4. Compute the lowest 6 eigenvalues and eigenfunctions for the Ginzburg-Landau double-well potential

$$V(x) = -\frac{5}{2}x^2 + \frac{1}{4}x^4$$

Plot these eigenfunctions and state the energy for each. Do you think they are correct or not? Why?