QUANTUM MECHANICS I

PHYS 516

Problem Set # 4

Distributed: Feb. 7, 2011

Due: Feb. 14, 2011

1. Elementary Wave Mechanics: Place a particle of mass m in a one-dimensional box (infinitely deep potential) of length L. Set $m = \hbar = 1$ and $L = 10$.

a. Compute the lowest 5 eigenstates and their energy eigenvalues.

b. Normalize the eigenfunctions correctly. (What does that mean?)

c. Plot the five lowest eigenvectors.

2. Matrix Mechanics - Discretization and Analytic Solution: Discretize the Schrödinger equation for a particle in a box of length $L = 10$. Set $m = \hbar = 1$. State explicitly what your step size is and the size of the matrix you are diagonalizing.

a. Show that the discrete equation consists of a tridiagonal matrix. What is this matrix?

b. Plot the eigenvalue spectrum for this discrete problem with the analytic solutions you computed in Problem #1. You can use the eigenfunctions we constructed in Class. Point out where the numerical and analytic solutions begin to diverge.

c. Plot the five lowest eigenfunctions.

d. How do you normalize these eigenfunctions?

3. Matrix Mechanics - Discretization and Numerical Solution: Discretize the Schrödinger equation for a particle in a box of length $L = 10$. Set $m = \hbar = 1$. State explicitly what your step size is and the size of the matrix you are diagonalizing.

a. Sort and plot all energy eigenvalues.

b. Compare to the eigenvalues that can be obtained analytically.

c. Which eigenfunctions *might* you believe and which would you definitely *not* believe?

d. Plot the "lowest" five eigenfunctions (this means the eigenfunctions belonging to the five smallest eigenvalues).

e. Compare these eigenvectors with the analytically available eigenvectors (c.f., your solutions to Problem #1). What are the similarities and differences? What can you say about signs and normalization? How do you normalize the numerical eigenfunctions so you can compare them with the analytic eigenfunctions?

4. Wave Mechanics - Harmonic Oscillator and Analytic Solution: Place a particle of mass m in a one-dimensional harmonic oscillator potential. Set $m = k = \hbar = 1$.

a. Write down the energy eigenvalues for the five lowest states.

b. Write down the eigenfunctions with the five lowest energy engenvalues. Be sure the get the normalization correct.

c. Plot the five lowest eigenfunctions $\psi_n(x)$.

5. Matrix Mechanics - More discretization and Numerical Solution: Discretize the Schrödinger equation for a particle in a one-dimensional harmonic oscillator potential. Set $m = k = \hbar = 1$. State explicitly what your step size is and the size of the matrix you are diagonalizing.

a. Sort and plot all energy eigenvalues from small to large.

b. Compare to the eigenvalues that can be obtained analytically (c.f., Problem # 4).

c. Which eigenfunctions *might* you believe and which would you definitely *not* believe?

d. Plot the five "lowest" eigenfunctions (this means the eigenfunctions belonging to the five smallest eigenvalues).

e. Compare these eigenvectors with the analytically available eigenvectors (c.f., your solutions to Problem #4). What are the similarities and differences? What can you say about signs and normalization? How do you normalize the numerical eigenfunctions so you can compare them with the analytic eigenfunctions?