QUANTUM MECHANICS I

PHYS 516

Problem Set # 2: 3-Dimensional Oscillators

Distributed: Jan. 19, 2011

Due: Jan. 28, 2011

1. Harmonic Motion in 3D: A particle of mass m is placed in a potential of the form $V(x, y, z) = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 + \frac{1}{2}k_z z^2$.

a. Show that Schrödinger's equation separates in Cartesian coordinates.

b. Write down the energy eigenvalue and the wavefunction when there is 1 excitation in the x direction, 2 in the y direction, and 3 in the z direction.

c. $E_{n_1,n_2,n_3} =$?

d. Suppose $\omega_1 = \omega_2 = 1.0$ and $\omega_3 = 1.1$. Determine the energy spectrum when 3 or fewer excitations are present. If (when) there is an energy degeneracy, identify the number of states in the degenerate multiplet.

e. If all spring constants are the same, what is the degeneracy of the state containing N quanta? (Hint: this is the same as the Bose-Einstein counting problem.)

2. Molecules: The linear molecule $ABBA$ oscillates in one dimension. The masses are $M_A = M$, $M_B = 2M$. The interaction is between only adjacent atoms and is represented by linear springs with spring constants $k_{AB} = k_{BA} = k$ and $k_{BB} = 3k$.

a. Describe the classical normal modes.

b. For each, what is the energy?

c. Quantize the vibrations of this molecule.

d. Write down the expression for the quantum vibrational energies.

3. Lattices: A simple, very short linear lattice consists of only three atoms. Each has mass m. The atoms are connected to each other with springs of spring constant k. The two atoms at the ends of this chain are connected to brick walls with springs of spring constant k (there is a total of 4 identical springs).

a. Determine the resonance frequencies.
b. Describe the three normal modes.
c. Write down the quantum mechanical hamiltonian.
d. Write down the energy spectrum.

4. Nuclear Physics: Fill in the table:

Harmonic Oscillator		Angular Momentum		
$N = n_1 + n_2 + n_3$	Degeneracy	L	Notation	Degeneracy
0	1	0	S	1
1	3	1	P	3
2	6	2	D	5
		0	S	1
3	10	3	F	7
		1	P	3
4				
5				
6				

b. Plot the energy level spectrum generated by the perturbed harmonic oscillator hamiltonian

$$\mathcal{H} = (N + \frac{3}{2})E_0 + \alpha \mathbf{L} \cdot \mathbf{L}$$

with $E_0 = 1.0\text{MeV}$ and $\alpha = 0.1$ MeV. (The spectrum of $\mathbf{L} \cdot \mathbf{L}$ is $L(L+1)$.)

5. Particle Physics: The 10 states in the decuplet multiplet, $|n_1, n_2, n_3\rangle$, with $n_i \geq 0$, $n_1 + n_2 + n_3 = 3$, can be assigned to the particles (See http://en.wiki- pedia.org/wiki/Baryon, Table $J^P = \frac{3}{2}^+$ Baryons):

Particle	Charge	Mass(MeV)	$n_1 n_2 n_3$
Ω^-	$-$	1672.45 ± 0.29	003
Ξ^{*-}	$-$	1535.0 ± 0.6	102
Ξ^{*0}	0	1531.80 ± 0.32	012
Σ^{*-}	$-$	1387.2 ± 0.5	201
Σ^{*0}	0	1383.7 ± 1.0	111
Σ^{*+}	$+$	1382.8 ± 0.4	021
Δ^-	$-$	1232 ± 1	300
Δ^0	0	1232 ± 1	210
Δ^+	$+$	1232 ± 1	120
Δ^{++}	$++$	1232 ± 1	030

a. Propose a simple model (linear in n_1, n_2, n_3) to describe the masses.

b. Carry out a χ^2 test on this model. Reject or "Accept" (i.e., Fail to Reject) your fit to the data. Give me a "story".

c. How well does your model describe "the three fundamental quarks"?