QUANTUM MECHANICS I

PHYS 516

Problem Set # 1
Distributed: Jan. 7, 2011
Due: Jan. 17, 2011

The Frobenius Method is a classical workhorse for finding solutions of relatively simple ordinary differential equations. (The link is to the course website: http://www.physics.drexel.edu/~bob/PHYS516_11/Frobenius.pdf). Here are two examples:

$$\left(\frac{d^2}{dr^2} + \frac{A}{r^2} + \frac{B}{r} + C\right) R(r) = 0$$

$$\left(\frac{d^2}{dr^2} + \frac{D}{r^2} + E + Fr^2\right) R(r) = 0$$

The first is a general form for the radial part of the quantum wavefunction for a Coulomb potential. The second is a general form for the radial part of the quantum wavefunction for a harmonic oscillator potential.

1. Use the Frobenius method to determine the quantization condition for square-integrable radial functions $\int_0^\infty R^2(r)dr < \infty$:

Coulomb: on the coefficients A, B, C;

Harmonic Oscillator: on the coefficients D, E, F.

2. For the Coulomb problem in three dimensions determine the coefficients A, B, C in the

a. relativistic case (Klein-Gordan Equation)

b. nonrelativistic case (NR Schrödinger Equation).

3. Determine the coefficients D, E, F for the nonrelativistic harmonic oscillator in

i. one dimension

ii. two dimensions

iii. three dimensions

4. Compute the energy of an electron (in eV) in the most tightly bound state about each of these nuclei:

	$Z = 1$ Proton	$Z = 26$ Iron Nucleus	$Z = 82$ Lead Nucleus
N.R. Schrödinger Equation	-13.58		
Relativistic Klein-Gordon Eq.			

In computing the relativistic energy, subtract off the electron rest energy mc^2 and enter $E - mc^2$ in the table above.

In this problem, if there are any surprises, explain:

a. What?

b. Where?

c. Why?

d. What does it mean?

e. What to do about it?

5. The expression for the energies of the "relativistic hydrogen atom" derived from the Klein Gordan equation is

$$E = \frac{mc^2}{\sqrt{1 + \left(\frac{\alpha}{N(\alpha)}\right)^2}}$$

where $N(\alpha) = n + \frac{1}{2} + \sqrt{(l + \frac{1}{2})^2 - \alpha^2}$ and α is the fine structure constant, $\alpha = 1/137.03611 \simeq .007$. Here $n = 0, 1, 2, ...$ is the "radial" quantum number and $l = 0, 1, 2, ...$ is the orbital angular momentum quantum number. The principle quantum number N (the one you remember from your undergraduate course) is $N = n + l + 1$.

a. Expand E in powers of α up to, and including, the sixth degree (use Maple or/and go crazy!). Explain the leading (zeroth order) term. Compare the second order term (the one containing α^2) with the expression for the nonrelativistic energies of the hydrogen atom. Express both in terms of N. Write down the fourth order terms. Express these energies in terms of N and l. These are close to the usual "relativistic corrections" to the nonrelativistic hydrogen atom.

b. Optional: For a single electron around a nucleus of charge Ze the energy eigenvalues are obtained from E above by the substitution $\alpha \to Z\alpha$. What happens when Z gets "too big"? How big is "too big"? What do you do then?