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> ###  Prof.R.  Gilmore  Quantum Mechanics 1:
> ###  Constructs the normalized hydrogen radial wavefunctions
> ###  Under (int(RX(n,l,r)*R(n,l,r)*r^2,r=0..infinity)=1
>
>
>
>
> restart;
> with(orthopoly);    ###  calls in useful orthog polys, like the Laguerres
```

$$[G, H, L, P, T, U]$$

```
> L(5,x);             ###  for example, it works
```

$$1 - 5\,x + 5\,x^2 - \frac{5}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{120}x^5$$

```
> g(5,0,x):=L(5,x);   ###  begins process for computing Associated Laguerrs
  polys
```

$$g(5, 0, x) := 1 - 5\,x + 5\,x^2 - \frac{5}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{120}x^5$$

```
> for i from 1 to 7 do
  g(5,i,x):=diff(g(5,i-1,x),x):print(5,i-1,g(5,i-1,x)):od:    #(it works)
```

$$5, 0, 1 - 5\,x + 5\,x^2 - \frac{5}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{120}x^5$$

$$5, 1, -5 + 10\,x - 5\,x^2 + \frac{5}{6}x^3 - \frac{1}{24}x^4$$

$$5, 2, 10 - 10\,x + \frac{5}{2}x^2 - \frac{1}{6}x^3$$

$$5, 3, -10 + 5\,x - \frac{1}{2}x^2$$

$$5, 4, 5 - x$$

$$5, 5, -1$$

$$5, 6, 0$$

```
> for j from 0 to 10 do g(j,0,x):=L(j,x); for i from 1 to j+2 do
  g(j,i,x):=diff(g(j,i-1,x),x):od:od:##  Lots of aqssoc lag.
> for n from 1 to 5 do for l from 0 to n-1 do R(n,l,x):=
  g(n+l,2*l+1,x)*exp(-x/2)*(x)^l:RR(n,l,r):=subs(x=2*r/n,%): N(n,l):=int(r^
  2*RR(n,l,r)^2,r=0..infinity):RX(n,l,r):=RR(n,l,r)/sqrt(N(n,l)); print(n,l
  ,RX(n,l,r)):od:od:   ### constructs normalized hydrogenic radial
  wavefunctions
```

$$1, 0, -2\,e^{(-r)}$$

$$2,\ 0,\ \frac{1}{4}(-2+r)\,\mathbf{e}^{\left(-\frac{1}{2}r\right)}\sqrt{2}$$

$$2,\ 1,\ -\frac{1}{12}\mathbf{e}^{\left(-\frac{1}{2}r\right)}r\sqrt{6}$$

$$3,\ 0,\ \frac{2}{27}\left(-3+2r-\frac{2}{9}r^2\right)\mathbf{e}^{\left(-\frac{1}{3}r\right)}\sqrt{3}$$

$$3,\ 1,\ \frac{1}{81}\left(-4+\frac{2}{3}r\right)\mathbf{e}^{\left(-\frac{1}{3}r\right)}r\sqrt{6}$$

$$3,\ 2,\ -\frac{2}{1215}\mathbf{e}^{\left(-\frac{1}{3}r\right)}r^2\sqrt{30}$$

$$4,\ 0,\ \frac{1}{16}\left(-4+3r-\frac{1}{2}r^2+\frac{1}{48}r^3\right)\mathbf{e}^{\left(-\frac{1}{4}r\right)}$$

$$4,\ 1,\ \frac{1}{480}\left(-10+\frac{5}{2}r-\frac{1}{8}r^2\right)\mathbf{e}^{\left(-\frac{1}{4}r\right)}r\sqrt{15}$$

$$4,\ 2,\ \frac{1}{1920}\left(-6+\frac{1}{2}r\right)\mathbf{e}^{\left(-\frac{1}{4}r\right)}r^2\sqrt{5}$$

$$4,\ 3,\ -\frac{1}{26880}\mathbf{e}^{\left(-\frac{1}{4}r\right)}r^3\sqrt{35}$$

$$5,\ 0,\ \frac{2}{125}\left(-5+4r-\frac{4}{5}r^2+\frac{4}{75}r^3-\frac{2}{1875}r^4\right)\mathbf{e}^{\left(-\frac{1}{5}r\right)}\sqrt{5}$$

$$5,\ 1,\ \frac{1}{1875}\left(-20+6r-\frac{12}{25}r^2+\frac{4}{375}r^3\right)\mathbf{e}^{\left(-\frac{1}{5}r\right)}r\sqrt{30}$$

$$5,\ 2,\ \frac{2}{65625}\left(-21+\frac{14}{5}r-\frac{2}{25}r^2\right)\mathbf{e}^{\left(-\frac{1}{5}r\right)}r^2\sqrt{70}$$

$$5, 3, \frac{1}{328125}\left(-8 + \frac{2}{5} r \right) \mathbf{e}^{\left(-\frac{1}{5} r \right)} r^3 \sqrt{70}$$

$$5, 4, -\frac{2}{4921875} \mathbf{e}^{\left(-\frac{1}{5} r \right)} r^4 \sqrt{70}$$

```
> for n from 1 to 5 do for l from 0 to n-1 do print(n,l,RX(n,l,r)):od:od:
```

$$1, 0, -2 \mathbf{e}^{(-r)}$$

$$2, 0, \frac{1}{4}(-2 + r) \mathbf{e}^{\left(-\frac{1}{2} r \right)} \sqrt{2}$$

$$2, 1, -\frac{1}{12} \mathbf{e}^{\left(-\frac{1}{2} r \right)} r \sqrt{6}$$

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```
> for n from 1 to 5 do for l from 1 to n-1 do
  over(n,l):=int(RX(n,l,r)*RX(n,l-1,r)*r*r^2,r=0..infinity):print(n,l,over
  (n,l)):od:od:
```

$$2,\ 1,\ -3\sqrt{3}$$

$$3,\ 1,\ -9\sqrt{2}$$

$$3,\ 2,\ -\frac{9}{2}\sqrt{5}$$

$$4,\ 1,\ -6\sqrt{15}$$

$$4,\ 2,\ -12\sqrt{3}$$

$$4,\ 3,\ -6\sqrt{7}$$

$$5,\ 1,\ -15\sqrt{6}$$

$$5,\ 2,\ -\frac{15}{2}\sqrt{7}\sqrt{3}$$

$$5,\ 3,\ -30$$

$$5,\ 4,\ \frac{-45}{2}$$

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>
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