QUANTUM MECHANICS I

PHYS 516

Problem Set # 4

Distributed: Mar. 3, 2010

Due: Mar. 15, 2010

1. Coherent States and Minimum Uncertainty: A particle in a harmonic oscillator potential is in a coherent state $|\alpha\rangle$, for which $|\alpha\rangle = e^{-\alpha^*\alpha/2}\, e^{\alpha a^\dagger}|0\rangle$ so that $a|\alpha\rangle = \alpha|\alpha\rangle$ (c.f., Ballentine pp. 541-548).

a. Compute $\overline{x} = \langle x\rangle = \langle\alpha|x|\alpha\rangle$.

b. Compute $\Delta x^2 = \langle(x-\overline{x})^2\rangle$.

c. Compute Δx.

d. Compute $\overline{p}, \Delta p^2, \Delta p$.

e. Compute $\Delta x \Delta p$.

2. Matrix Mechanics - Discretization: Discretize the Schrödinger equation for a particle in a box of length $L = 1$. Set $m = \hbar = 1$. State explicitly what your step size is and the size of the matrix you are diagonalizing.

a. Sort and plot all energy eigenvalues.

b. Compare to the eigenvalues that can be obtained analytically.

c. Which eigenfunctions *might* you believe and which would you definitely *not* believe?

d. Plot the "lowest" five eigenfunctions (this means the eigenfunctions belonging to the five smallest eigenvalues).

e. Compare these eigenvectors with the analytically available eigenvectors (c.f., Dicke & Wittke, Chap. 3). What are the similarities and differences? What can you say about signs and normalization?

3. Matrix Mechanics - More discretization: Discretize the Schrödinger equation for a particle in a harmonic oscillator potential. Set $m = k = \hbar = 1$. State explicitly what your step size is and the size of the matrix you are diagonalizing.

a. Sort and plot all energy eigenvalues from small to large.

b. Compare to the eigenvalues that can be obtained analytically.

c. Which eigenfunctions *might* you believe and which would you definitely *not* believe?

d. Plot the "lowest" five eigenfunctions (this means the eigenfunctions belonging to the five smallest eigenvalues) (c.f., Ballentine, Chap. 6).

e. Compute the matrix elements $\langle n+1|x|n\rangle$ and $\langle n+1|p|n\rangle$ for $n = 0$ using the numerical eigenvectors.

4. Wave Mechanics - Matrix Mechanics Comparison:

a. For the harmonic oscillator plot the five lowest eigenfunctions $\psi_n(x) = H_n(x)e^{-x^2/2}/\sqrt{2^n n!\sqrt{\pi}}$, $n = 0, 1, 2, 3, 4$.

b. Plot the numerical eigenfunctions from Problem 3**d.** on the same graph.

c. Say something clever about the sign and normalization problems that you encounter.

d. Compute the matrix elements $\langle n+1|x|n\rangle$ and $\langle n+1|p|n\rangle$ for $n = 0$ using the analytic eigenvectors. Compare with your answer to Problem 3**e.**.

5. Matrix Mechanics - More discretization: Discretize the Schrödinger equation for a particle in a bimodal potential $V(x) = \frac{1}{4}x^4 - \alpha x^2$. Set $m = k = \hbar = 1$. Set $\alpha = 2$. State explicitly what your step size is and the size of the matrix you are diagonalizing.

a. Sort and plot all energy eigenvalues from small to large.

b. Which eigenfunctions *might* you believe and which would you definitely *not* believe?

c. Plot the "lowest" six eigenfunctions (this means the eigenfunctions belonging to the six smallest eigenvalues) (c.f., Ballentine, Chap. 6).

d. Which of these eigenfunctions do you believe? Why?

6. Matrix Mechanics All Over Again: The one-dimensional potential $V(x)$ is symmetric and it is desired to compute the eigenvalues and eigenfunctions of a particle confined by this potential. Set $m = k = \hbar = 1$. Choose an interval $-L \leq x \leq +L$ and choose basis functions $\cos\left(\frac{k\pi x}{L}\right)$ for $k = 0, 1, 2, \cdots, N$ and $\sin\left(\frac{k\pi x}{L}\right)$ for $n = 1, 2, \cdots, N$.

A. Choose $V(x) = \frac{1}{2}x^2$.

a. What is your reasonable choice of L? What value of N are you using?

b. Compute the matrix elements $\langle k|H|k'\rangle$ in the cosine basis.

c. Compute the eigenvalues of this matrix.

d. Plot the four lowest eigenvectors.

e. Do you believe these eigenfunctions? Why/not?

f. If you don't believe them, what must you do to compute a more believable set? Do it.

g. Repeat these calculations in the sine basis.

h. Why don't you need to use the mixed sine/cosine basis?

B. Choose $V(x) = \frac{1}{4}x^4 - \alpha x^2$.

i. Repeat steps **a.** through **g.** for this bimodal potential.