QUANTUM MECHANICS I

PHYS 516

Problem Set # 3: 3-Dimensional Oscillators

Distributed: Feb 17, 2010

Due: Feb. 26, 2010

1. Particle Physics: The 10 states in the decuplet multiplet, $|n_1, n_2, n_3\rangle$, with $n_i \geq 0$, $n_1+n_2+n_3 = 3$, can be assigned to the particles (See http://en.wikipedia.org/wiki/Baryon, Table $J^P = \frac{3}{2}^+$ Baryons):

Particle	Charge	Mass(MeV)	$n_1 n_2 n_3$
Ω^-	$-$	1672.45 ± 0.29	003
Ξ^{*-}	$-$	1535.0 ± 0.6	102
Ξ^{*0}	0	1531.80 ± 0.32	012
Σ^{*-}	$-$	1387.2 ± 0.5	201
Σ^{*0}	0	1383.7 ± 1.0	111
Σ^{*+}	$+$	1382.8 ± 0.4	021
Δ^-	$-$	1232 ± 1	300
Δ^0	0	1232 ± 1	210
Δ^+	$+$	1232 ± 1	120
Δ^{++}	$++$	1232 ± 1	030

a. Propose a simple model (linear in n_1, n_2, n_3) to describe the masses.

b. Carry out a χ^2 test on this model. Reject or "Accept" (i.e., Fail to Reject) your fit to the data. Give me a "story".

c. How well does your model describe "the three fundamental quarks"?

2. Nuclear Physics: Fill in the table:

Harmonic Oscillator		Angular Momentum		
$N = n_1 + n_2 + n_3$	Degeneracy	L	Notation	Degeneracy
0	1	0	S	1
1	3	1	P	3
2	6	2	D	5
		0	S	1
3	10	3	F	7
		1	P	3
4				
5				
6				

b. Plot the energy level spectrum generated by the perturbed harmonic oscillator hamiltonian

$$\mathcal{H} = (N + \frac{3}{2})E_0 + \alpha \mathbf{L} \cdot \mathbf{L}$$

with $E_0 = 1.0$MeV and $\alpha = 0.1$ MeV. (The spectrum of $\mathbf{L} \cdot \mathbf{L}$ is $L(L+1)$.)

3. Molecules: The linear molecule $ABBA$ oscillates in one dimension. The masses are $M_A = M,\ M_B = 2M$. The interaction is between only adjacent atoms and is represented by linear springs with spring constants $k_{AB} = k_{BA} = k$ and $k_{BB} = 3k$.

a. Describe the classical normal modes.

b. For each, what is the energy?

c. Quantize the vibrations of this molecule.

d. Write down the expression for the quantum vibrational energies.

4. Lattices: A simple, very short linear lattice consists of only three atoms. Each has mass m. The atoms are connected to each other with springs of spring constant k. The two atoms at the ends of this chain are connected to brick walls with springs of spring constant k (there is a total of 4 identical springs).

a. Determine the resonance frequencies.

b. Describe the three normal modes.

c. Write down the quantum mechanical hamiltonian.

d. Write down the energy spectrum.