QUANTUM MECHANICS I

PHYS 516

Problem Set # 2
Distributed: Jan. 20, 2010
Due: Jan. 29, 2010

1. Expand the expression for the relativistic energies of the Coulomb problem

$$E(N,l) = \frac{mc^2}{\sqrt{1+\left(\frac{\alpha}{N(\alpha)}\right)^2}}$$

in powers of α to (and including) α^8. Express your results in terms of the principal quantun number N and the orbital angular momentum l. Here $N(\alpha) = n + \frac{1}{2} + \sqrt{(l+\frac{1}{2})^2 - \alpha^2}$, $N = n + l + 1$, and n is the "radial" quantum number, the number of nodes in the radial wave function. (**Hint: use Maple.**) Provide a physical interpretation of the first two terms.

2. Compute the energy change in eV. for an electron in state Nl in a nucleus with charge Z caused by the finite nuclear size:

Student	N	l	Z
Hayley Finley	1	0	2
Austen Groener	2	0	3
Frank Jones	2	1	7
Crystal Moorman	3	0	5
Allyson O'Brien	3	1	4
Erica Smith	4	0	26
Nicholas Smith	3	2	2

3. $\hat{a}$ is a nonhermitian operator with matrix elements $\langle n|\hat{a}|n'\rangle = \delta_{n,n'-1}\sqrt{n'}$, $0 \leq n, n' \ldots$.

a. Write down (part of) the matrix of $\hat{a}$.

b. What are the matrix elements of the hermitian conjugate operator $\hat{a}^\dagger$?

c. Write down (part of) the matrix of this operator.

d. Write down the matrices $\hat{a}\hat{a}^\dagger$ and $\hat{a}^\dagger\hat{a}$.

e. Write down the commutator $\left[\hat{a}, \hat{a}^\dagger\right]$.

f. Write down half the anticommutator: $\frac{1}{2}\left\{\hat{a}, \hat{a}^\dagger\right\}$.

g. What are the matrix elements of $\hat{a}$ in the coordinate representation: $\langle x|\hat{a}|x'\rangle$?

h. Repeat parts **b.** through **f.** in the coordinate representation.

i. Write down the normalized matrix elements $\langle x|n\rangle$ of the similarity transformation.