

22.6: Differential Equations

$$g_2(x)y'' + g_1(x)y' + g_0(x)y = 0$$

	y	$g_2(x)$	$g_1(x)$	$g_0(x)$
22.6.1	$P_n^{(\alpha, \beta)}(x)$	$1-x^2$	$\beta - \alpha - (\alpha + \beta + 2)x$	$n(n + \alpha + \beta + 1)$
22.6.2	$(1-x)^\alpha(1+x)^\beta P_n^{(\alpha, \beta)}(x)$	$1-x^2$	$\alpha - \beta + (\alpha + \beta - 2)x$	$(n+1)(n + \alpha + \beta)$
22.6.3	$(1-x)^{\frac{\alpha+1}{2}}(1+x)^{\frac{\beta+1}{2}} P_n^{(\alpha, \beta)}(x)$	1	0	$\frac{1}{4} \frac{1-\alpha^2}{(1-x)^2} + \frac{1}{4} \frac{1-\beta^2}{(1+x)^2}$ $+ \frac{2n(n + \alpha + \beta + 1) + (\alpha + 1)(\beta + 1)}{2(1-x^2)}$
22.6.4	$\left(\sin \frac{x}{2}\right)^{\alpha+\frac{1}{2}} \left(\cos \frac{x}{2}\right)^{\beta+\frac{1}{2}} P_n^{(\alpha, \beta)}(\cos x)$	1	0	$\frac{1-4\alpha^2}{16 \sin^2 \frac{x}{2}} + \frac{1-4\beta^2}{16 \cos^2 \frac{x}{2}}$ $+ \left(n + \frac{\alpha + \beta + 1}{2}\right)^2$
22.6.5	$C_n^{(\alpha)}(x)$	$1-x^2$	$-(2\alpha+1)x$	$n(n+2\alpha)$
22.6.6	$(1-x^2)^{\alpha-\frac{1}{2}} C_n^{(\alpha)}(x)$	$1-x^2$	$(2\alpha-3)x$	$(n+1)(n+2\alpha-1)$
22.6.7	$(1-x^2)^{\frac{\alpha+1}{4}} C_n^{(\alpha)}(x)$	1	0	$\frac{(n+\alpha)^2}{1-x^2} + \frac{2+4\alpha-4\alpha^2+x^2}{4(1-x^2)^2}$
22.6.8	$(\sin x)^\alpha C_n^{(\alpha)}(\cos x)$	1	0	$(n+\alpha)^2 + \frac{\alpha(1-\alpha)}{\sin^2 x}$
22.6.9	$T_n(x)$	$1-x^2$	$-x$	n^2
22.6.10	$T_n(\cos x)$	1	0	n^2
22.6.11	$\frac{1}{\sqrt{1-x^2}} T_n(x); U_{n-1}(x)$	$1-x^2$	$-3x$	n^2-1
22.6.12	$U_n(x)$	$1-x^2$	$-3x$	$n(n+2)$
22.6.13	$P_n(x)$	$1-x^2$	$-2x$	$n(n+1)$
22.6.14	$\sqrt{1-x^2} P_n(x)$	1	0	$\frac{n(n+1)}{1-x^2} + \frac{1}{(1-x^2)^2}$
22.6.15	$L_n^{(\alpha)}(x)$	x	$\alpha+1-x$	n
22.6.16	$e^{-x} x^{\alpha/2} L_n^{(\alpha)}(x)$	x	$\alpha+1$	$n + \frac{\alpha}{2} + 1 - \frac{\alpha^2}{4x}$
22.6.17	$e^{-x/2} x^{\alpha+1/2} L_n^{(\alpha)}(x)$	1	0	$\frac{2n+\alpha+1}{2x} + \frac{1-\alpha^2}{4x^2} - \frac{1}{4}$
22.6.18	$e^{-x^2/2} x^{\alpha+1/2} L_n^{(\alpha)}(x^2)$	1	0	$4n+2\alpha+2-x^2 + \frac{1-4\alpha^2}{4x^2}$
22.6.19	$H_n(x)$	1	$-2x$	$2n$
22.6.20	$e^{-x^2/2} H_n(x)$	1	0	$2n+1-x^2$
22.6.21	$He_n(x)$	1	$-x$	n

(*See page 11.