1. Coherent States and Minimum Uncertainty: A particle in a harmonic oscillator potential is in a coherent state |α⟩, for which |α⟩ = e^{−α^∗α/2} e^{αa^†}|0⟩ so that a|α⟩ = α|α⟩ (c.f., Ballentine pp. 541-548).
   a. Compute x = ⟨x⟩ = ⟨α|x|α⟩.
   b. Compute Δx^2 = ⟨(x−x)^2⟩.
   c. Compute Δx.
   d. Compute p, Δp^2, Δp.
   e. Compute ΔxΔp.

2. Matrix Mechanics - Discretization: Discretize the Schrödinger equation for a particle in a box of length L = 1. Set m = ¯h = 1. State explicitly what your step size is and the size of the matrix you are diagonalizing.
   a. Sort and plot all energy eigenvalues.
   b. Compare to the eigenvalues that can be obtained analytically.
   c. Which eigenfunctions might you believe and which would you definitely not believe?
   d. Plot the “lowest” five eigenfunctions (this means the eigenfunctions belonging to the five smallest eigenvalues).
   e. Compare these eigenvectors with the analytically available eigenvectors (c.f., Dicke & Wittke, Chap. 3). What are the similarities and differences? What can you say about signs and normalization?

3. Matrix Mechanics - More discretization: Discretize the Schrödinger equation for a particle in a harmonic oscillator potential. Set m = k = ¯h = 1. State explicitly what your step size is and the size of the matrix you are diagonalizing.
   a. Sort and plot all energy eigenvalues.
   b. Compare to the eigenvalues that can be obtained analytically.
   c. Which eigenfunctions might you believe and which would you definitely not believe?
d. Plot the “lowest” five eigenfunctions (this means the eigenfunctions belonging to the five smallest eigenvalues) (c.f., Ballentine, Chap. 6).

4. Wave Mechanics - Matrix Mechanics Comparison:
   a. For the harmonic oscillator plot the five lowest eigenfunctions \( \psi_n(x) = H_n(x)e^{-x^2/2}/\sqrt{2^n n!\sqrt{\pi}}, \ n = 0, 1, 2, 3, 4. \)
   b. Plot the numerical eigenfunctions from Problem 3d. on the same graph.
   c. Say something clever about the sign and normalization problems that you encounter.

5. Lattices and Unit Cells:
   A linear harmonic lattice consists of \( N \) unit cells, each with two atoms, one of mass \( m \), the other of mass \( M \). The intra-unit cell spring has constant \( K \) and the spring between unit cells has value \( k \). Plot the dispersion relation \( \omega(j) \) for this lattice for \( -\frac{N}{2} \leq j \leq +\frac{N}{2} \). (Choose the ratios \( M/m \) and \( K/k \) to have different values in the range 2 to 5. Set boundary conditions of your own choosing, assuming \( N \) large.)

6. Zero Point Energy:
   A linear harmonic lattice consists of \( N \) identical atoms of mass \( m \) interacting via nearest neighbor coupling with springs of spring constant \( k \). The two end masses are connected with identical springs to “brick walls”. The distance between the walls is \( L \) (c.f., Ballentine, pp. 533-539).
   a. Compute the ground state (zero point) energy.
   b. Compute the mass equivalence of this ground state energy.
   c. Compute the ground state (zero point) energy density (energy per lattice spacing).
   b. Compute the mass equivalence of this zero point energy density.