1. Compute the radial matrix element for the transition \((N, l) \rightarrow (N-1, l-1)\) in the hydrogen atom. Assume that \(l\) “saturates” \(N\): \(l = N - 1\).

2. Solve the dimensionless one-dimensional quantum harmonic oscillator using the Frobenius method. Derive the recurrence relation for the (Hermite) polynomial functions and construct these polynomials to fourth order.

3. Use the recurrence relations on the algebraic and the geometric arguments of the Hermite polynomials to prove the following:

\[
\frac{1}{\sqrt{2}} \left( x + \frac{d}{dx} \right) \psi_n(x) = \sqrt{n} \psi_{n-1}(x)
\]
\[
\frac{1}{\sqrt{2}} \left( x - \frac{d}{dx} \right) \psi_n(x) = \sqrt{n+1} \psi_{n+1}(x)
\]

4. \(\psi(x, t) = a_n(t)\psi_n(x) + a_{n+1}(t)\psi_{n+1}(x), \quad |a_n(t)|^2 + |a_{n+1}(t)|^2 = 1.\)
   
a. \(\langle x \rangle(t) = ?\)
   
b. \(\langle p \rangle(t) = ?\)
   
c. What are these expectation values if \(\psi(x, t) = a_n(t)\psi_n(x) + a_{n+2}(t)\psi_{n+2}(x)\)? Why?