1. The energy of a photon emitted in the transition of the hydrogen atom from a state with principal quantum number \( N_2 \) to a state with principal quantum number \( N_1 \) is \( \Delta E = \frac{1}{2} mc^2 \alpha^2 \left( \frac{1}{N_1^2} - \frac{1}{N_2^2} \right) \). Several series of transitions have been studied since “antiquity”:
  - Lyman: \( N_2 \rightarrow N_1 = 1 \)
  - Balmer: \( N_2 \rightarrow N_1 = 2 \)
  - Paschen: \( N_2 \rightarrow N_1 = 3 \)
  - Brackett: \( N_2 \rightarrow N_1 = 4 \)
  - Pfund: \( N_2 \rightarrow N_1 = 5 \)

  Each series has a sequence of lines, labeled as follows: \( \alpha : N_2 = N_1 + 1 \), \( \beta : N_2 = N_1 + 2 \), \( \gamma : N_2 = N_1 + 3 \), etc.

  a. Compute the energy, frequency, and wavelength for the Lyman \( \alpha \), Lyman \( \beta \), and Balmer \( \alpha \) lines.
  b. Compute the limits (\( N_2 \rightarrow \infty \)) for the Lyman and Balmer series.
  c. Sketch what you see in a spectrograph (Lyman series).
  d. Compute the energy, frequency, and wavelength of a photon emitted in the transition: \( N_2 = 101 \rightarrow N_1 = 100 \). What type of equipment is needed to detect such a photon (cosmic ray detector, X-ray detector, UV, visible, IR, far IR, picowave, microwave, esp, . . . )?

2. The mass \( m \) that appears in the Schrödinger equation is actually the proton-electron reduced mass. Starting from the Hamiltonian

\[
H = \frac{P_p^2}{2M_p} + \frac{P_e^2}{2m_e} - \frac{e^2}{|R_p - R_e|}
\]

move to center of mass and relative coordinates to show

\[
H = \frac{P_{T}^2}{2M_T} + \frac{P_r^2}{2m_r} - \frac{e^2}{|r|}
\]

a. Identify the relation between the terms in Equ. \((2)\) and Equ. \((1)\).
b. Look up the masses (or rest energies) of $e^-$ and $p^+$, including estimated errors. Cite your sources. Recompute the Lyman $\alpha$ energy, frequency, and wavelength using the proper reduced mass. Include errors in your estimates.

c. Compute the energy, frequency, and wavelength for the Lyman $\alpha$ line in deuterium.

d. A hydrogen cloud contains an $H$ to $D$ ratio of 1000 : 1. At what temperature can the Lyman $\alpha$ doublet no longer be resolved? Use $\tau = 10^{-8}$ sec.

3. Compute the energy level spectrum of the electron in the Coulomb potential established by the proton using the Klein-Gordan equation. You will find something like

$$E(N,l) = \frac{mc^2}{\sqrt{1 + f(n_r, l)^2}}$$

where $f(n_r, l) = \frac{\alpha}{n_r^2 + (l + \frac{1}{2})^2 - \alpha^2}$.

a. How does this expression change in a nucleus of charge $Ze$?

b. At what point (Z value) does this theory break down? Why?

c. Express the energy in terms of the principal quantum number $N = n_r + l + 1$. Then expand this expression in powers of $\alpha^2$.

- $(\alpha^2)^0$ show this
- $(\alpha^2)^1$ N.R Schrödinger spectrum show this
- $(\alpha^2)^2$ first relativistic corrections derive them
- $(\alpha^2)^3$ next higher order corrections derive them

d. Write down the relativistic corrections to the states $N,l$ to order $\alpha^4$.

e. Compute the splitting between the $2s$ and $2p$ states. Compute the splitting between the $3s$, $3p$ and $3d$ states. Compare these energies with the $2s - 3s$ splitting.

4. Find the mean radius of the $\pi^-$ (spin = 0) in the state with $n_r = 0$, $N = l + 1$. 