Mathematical Physics II

PHYS 502

Problem Set # 5
Distributed January 29, 2016
Due February 5, 2016

1. Compute parameters for the five probability distribution functions discussed in class so that

$$\langle x \rangle = \overline{x} = 0 \qquad \langle (x-\overline{x})^2 \rangle = (\Delta x)^2 = \sigma^2 = 1$$

To recall, these PDFs are: Gaussian $\simeq e^{-\alpha x^2}$; double-sided exponential $\simeq e^{-\alpha |x|}$; boxcar: constant for $-a \leq x \leq +a$, 0 outside; parabolic $a - bx^2$; triangular. Make sure they are properly normalized.

2. Dust off your favorite QRNG (tell me by name what it is). Apply it to the parabolic PDF using Monte Carlo methods. Bin the results in a histogram, plot the histogram, compare the analytic and the numerical results, and scream EUREKA! if the results warrant it.

3. Construct the CDF (cumulative distribution function) for the triangular PDF. Use your favorite QRNG to build up a probability distribution using this method (i.e., map the output of the RNG from the y axis down to the x axis). Bin the results in a histogram, plot the histogram, compare the analytic and the numerical results, and scream EUREKA! if the results warrant it.

4. Go back to the parabolic distribution and numerically build up a χ^2 distribution for 9 degrees of freedom. Use enough data so that your binned plot looks fairly smooth. Describe what you did in sufficient detail so that anybody else could repeat your work.

5. Using this Result: You make a series of 11 measurements. The x values are integers with no measurement error. The y coordinates have measurement errors following a parabolic PDF with $\sigma^2 = 0.5$. Find the best fit line and use the χ^2 distribution computed in Problem #4 to assess how confident you are of the linear model that is best fit to the data. Express your result quantitatively in a way that would make you appear as a "professional statistician" (*i.e.*, eminently employable) to the rest of the world.

x	10.0	8.0	13.0	9.0	11.0	14.0	6.0	4.0	12.0	7.0	5.0
y	8.04	6.95	7.58	8.81	8.33	9.96	7.24	4.26	10.84	4.82	5.68