Mathematical Physics II

PHYS 502

Midterm Exam, February 12, 2016

1. Scaling: Tests of the number of iterations that the Page Rank algorithm undergoes before convergence gave the first two lines below.

Test	n=# links	# Iterations
1.	160×10^6	45
2.	322×10^6	52
3.	10^9	

The first two lines suggest that the number of iterations obeys a logarithmic scaling law depending on the number of links. How many iterations do you expect to have to make to converge for a network with a billion links?

2. Masses: It has been discovered that three bodies of masses m_1, m_2, m_3 undergo a three-body interaction. Construct three invariant functions of the three masses with the property that each function has the dimensions of mass. Choose the functions $f_i(m_1, m_2, m_3)$ with the property that

$$\begin{aligned} f_1 &\xrightarrow{L} m_1 \\ f_2 &\xrightarrow{L} m_2 \\ f_3 &\xrightarrow{L} m_3 \end{aligned}$$

Here L is the limit $m_1 \gg m_2 \gg m_3$.

3. Experimental: A point source of light at the origin (*e.g.*, a star) is processed by an optical device and its image on a focal plane is a gaussian distribution $psf(\mathbf{x}) = \frac{\sqrt{det|g_{ij}|}}{(2\pi)^{2/2}} exp\left(-\frac{1}{2} g_{ij} x^i x^j\right)$. Here psf stands for 'point spread function'. The point spread function has a covariance matrix given by $\langle x^i x^j \rangle = g^{ij}$. An extended source with unknown intensity distribution $S(\mathbf{x})$ is imaged by this device. The image is also a gaussian distribution with covariance matrix $\langle x^i x^j \rangle = I^{ij}$.

a. Show that the image is a convolution of the source with the point spread function of the optical device.

b. What is the covariance matrix of the source?

4. Quantum Gravity: It is all the rage these days to construct a relativistically (c) correct quantum ($\hbar$) theory of gravity (G).

a. Construct an expression for the characteristic size in such a theory.

b. Use the measured values of $c = 3 \times 10^8$ m/sec. $\hbar = 1 \times 10^{-34}$kg.m^2/sec. and $G = 7 \times 10^{-11}$m^3/kg. sec.^{2}, precise to about 0 decimal points, to estimate this size.