METHODS of MATHEMATICAL PHYSICS

PHYS 324

Problem Set # 3

Distributed: January 28, 2014

Due: February 4, 2014

1. Particle in a Box — Analytic: A particle of mass m is placed inside a one-dimensional well of length L. Schrödinger's equation for this particle is

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = E\psi(x)$$

with boundary conditions $\psi(0) = 0,\ \psi(L) = 0$.

a. Compute the eigenvalues and eigenfunctions analytically.

b. Plot the lowest five eigenfunctions assuming $m = \hbar = 1$ and $L = 10$.

2. Particle in a Box — Numerical: Divide the length $L = 10$ into 100 equal-length intervals of length $\Delta = L/100$. Assume the value of the wavefunction at discrete point with x-value $x_i = i\Delta$ is ψ_i, so that $\psi_i = 0$ for $i = 0,\ i = 100$.

a. Approximate the second derivative operator in Schrödinger's equation by a matrix.

b. Approximate the Schrödinger equation as a matrix eigenvalue equation. Use $m = \hbar = 1$.

c. Find the eigenvalues and eigenvectors.

d. Compare the five lowest eigenvalues computed numerically with those computed analytically in Problem #**1.a.**

e. Plot the five lowest eigenvectors computed numerically (in **c.**) with those computed analytically and plotted in Problem #**1.b.**