

# TOPICS in MATHEMATICAL PHYSICS

## PHYS 324

### Problem Set # 2

**Distributed: January 16, 2014**

**Due: January 23, 2014**

**1. Eigenvalues and Eigenvectors:** Here is a  $5 \times 5$  matrix of a type (tridiagonal) that occurs ridiculously often in Physics:

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

Find its eigenvalues and eigenvectors.

**2. Coupled Oscillations:** Two rigid walls are a distance  $6L$  apart. Between these walls there are 5 particles, each of mass  $m$ . Each mass is connected to its two nearest neighbors by a spring of spring constant  $k$ , and the outside masses are connected to the rigid walls. The five masses are constrained to move along the  $x$  axis.

- a. Draw a picture.
- b. Introduce coordinates  $x_i$  to represent the displacement of the  $i^{\text{th}}$  mass from its equilibrium position.
- c. Write down an expression for the kinetic energy.
- d. Write down an expression for the potential energy.
- e. Write down the five Euler Lagrange equations of motion.
- f. Write these equations down in matrix form.
- g. Look at these matrices.
- h. Did a lightbulb go off? If 'Y', write down the solution using the result of the first problem. If 'N', ask in class "What's going on?"
- i. Draw a picture of the three lowest normal modes. If you don't know what 'draw a picture...' means, ask in class.