TOPICS in MATHEMATICAL PHYSICS

PHYS 324

Diagnostic Exam Results
Distributed: January 6, 2014

Instructions: Here are summary results of the Diagnostic Test. The numbers in parentheses are the number of students who made a serious attempt at the problem.

1. (12) Evaluate the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$.

2. (15) Solve the simultaneous linear equations for the values of x and y: $x + 2y = 3$ and $4x + 5y = 6$. Verify your answer.

3. (10) Compute the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$.

4. (3) Compute the eigenvalues and eigenvectors of the real symmetric matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

5. (0) Here is an equation for simple harmonic motion: $\frac{d^2x}{dt^2} = -\omega^2 x$. What is the eigenvalue of the second derivative operator $\frac{d^2}{dt^2}$? What are the two independent eigenvectors?

6. (7) The equations of motion for a simple harmonic oscillator are $\frac{dx}{dt} = p/m$ and $\frac{dp}{dt} = -kx$. Write these two first order equations as a single second order equation in the position coordinate x.

7. (1) Find the solutions of $\frac{d^2y(x)}{dx^2} = \text{cst}.y(x)$ that vanish at $x = 0$ and at $x = L$: $y(0) = y(L) = 0$. For each solution, find the value of cst.

8. (6) $y = \arctan(x) = \tan^{-1}(x)$ and $dy = \frac{dx}{1+x^2}$. Evaluate $\int_0^1 \arctan(x)\, dx$ using integration by parts.

9. (2) Evaluate the integral $\int_{-\infty}^{+\infty} \frac{x}{1+x^2} dx$ by symmetry. Evaluate the integral $\int_{-a}^{+a} xe^{-x^2} dx$ by symmetry.

10. (2) Evaluate the integral $\int_0^{+\infty} e^{-ax}\cos(bx)dx$ using the expression for $\cos(bx) = \frac{1}{2}(e^{+ibx} + e^{-ibx})$.

11. (11) $f(x,y,z) = xy^2z^3$. $\partial f/\partial x = ?$ $\partial f/\partial y = ?$ $\partial f/\partial z = ?$

12. (9) $f(x,y,z) = x^2y + y^2z + z^2x$. $\nabla f(x,y,z) = \mathrm{grad} f = ?$

13. (1) $f(x,y,z) = x^2y + y^2z + z^2x$. Evaluate the volume integral of $f(x,y,z)$ over the cube with unit edge length centered at $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$.

14. (0) $f(x,y,z) = x^2y + y^2z + z^2x$. Evaluate the volume integral of $f(x,y,z)$ over the sphere with unit radius centered at the origin. Use symmetry.

15. (3) $\mathbf{F}(x,y,z) = x^2y\mathbf{i} + y^2z\mathbf{j} + z^2x\mathbf{k}$.
$\nabla \times \mathbf{F}(x,y,z) = \mathrm{curl}\mathbf{F}(x,y,z) = ?$
$\nabla \cdot \mathbf{F}(x,y,z) = \mathrm{div}\mathbf{F}(x,y,z) = ?$

16. (0) Show that the function $f(t) = \frac{1}{1-ae^{i\omega t}}$ is periodic: $f(t+T) = f(t)$. Find T.

17. (0) Compute the Fourier expansion of the function $f(t) = \frac{1}{1-ae^{i\omega t}}$ ($|a| < 1$). Find the power spectrum of this function.

18. (0) $z = 3+4i$. Write the complex number z in (a) polar form $z = re^{i\theta}$; (b) in the form of a 2×2 matrix.

19. Fill in the blanks:

Ability	C	C^{++}	Python	FORTRAN	Maple	Mathematica	MatLab	LaTeX
Clueless	6	3	4	12	2	10	6	6
Maybe	5	5	2	1	9	3	4	4
Fluent	2	5	7		2		3	3

20. Where are you weak? What do you (think you) need to learn?

Linear Algebra	1	Eigenstuff	7
Vector Fields	1	Vector Operators	2
Geometry/Trigonometry		Computer aids for calculations	2
Special Functions	8	Complex Variables	3
Writing		Fourier Stuff	8
How to find webstuff	1	How to evaluate webstuff	1
Probability/Statistics	4	Other (specify)	3