

# TOPICS in MATHEMATICAL PHYSICS

## PHYS 324

### Diagnostic Exam

**Distributed: January 7, 2014**

**Instructions: DO NOT PUT YOUR NAME ON THE TEST BOOK.** Instead, place an identifier that you will not fail to recognize at the end of the Quarter. Answer the questions to the best of your ability.

1. Evaluate the inverse of the matrix  $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ .
2. Solve the simultaneous linear equations for the values of  $x$  and  $y$ :  $x + 2y = 3$  and  $4x + 5y = 6$ . Verify your answer.
3. Compute the eigenvalues of the matrix  $\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ .
4. Compute the eigenvalues and eigenvectors of the real symmetric matrix  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ .
5. Here is an equation for simple harmonic motion:  $\frac{d^2x}{dt^2} = -\omega^2x$ . What is the eigenvalue of the second derivative operator  $\frac{d^2}{dt^2}$ ? What are the two independent eigenvectors?
6. The equations of motion for a simple harmonic oscillator are  $\frac{dx}{dt} = p/m$  and  $\frac{dp}{dt} = -kx$ . Write these two first order equations as a single second order equation in the position coordinate  $x$ .
7. Find the solutions of  $\frac{d^2y(x)}{dx^2} = \text{cst.}y(x)$  that vanish at  $x = 0$  and at  $x = L$ :  $y(0) = y(L) = 0$ . For each solution, find the value of cst.
8.  $y = \arctan(x) = \tan^{-1}(x)$  and  $dy = \frac{dx}{1+x^2}$ . Evaluate  $\int_0^1 \arctan(x) dx$  using integration by parts.
9. Evaluate the integral  $\int_{-\infty}^{+\infty} \frac{x}{1+x^2} dx$  by symmetry. Evaluate the integral  $\int_{-a}^{+a} xe^{-x^2} dx$  by symmetry.

10. Evaluate the integral  $\int_0^{+\infty} e^{-ax} \cos(bx) dx$  using the expression for  $\cos(bx) = \frac{1}{2}(e^{+ibx} + e^{-ibx})$ .

11.  $f(x, y, z) = xy^2z^3$ .  $\partial f/\partial x = ?$   $\partial f/\partial y = ?$   $\partial f/\partial z = ?$

12.  $f(x, y, z) = x^2y + y^2z + z^2x$ .  $\nabla f(x, y, z) = \text{grad} f = ?$

13.  $f(x, y, z) = x^2y + y^2z + z^2x$ . Evaluate the volume integral of  $f(x, y, z)$  over the cube with unit edge length centered at  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ .

14.  $f(x, y, z) = x^2y + y^2z + z^2x$ . Evaluate the volume integral of  $f(x, y, z)$  over the sphere with unit radius centered at the origin. Use symmetry.

15.  $\mathbf{F}(x, y, z) = x^2y\mathbf{i} + y^2z\mathbf{j} + z^2x\mathbf{k}$ .

$\nabla \times \mathbf{F}(x, y, z) = \text{curl}\mathbf{F}(x, y, z) = ?$

$\nabla \cdot \mathbf{F}(x, y, z) = \text{div}\mathbf{F}(x, y, z) = ?$

16. Show that the function  $f(t) = \frac{1}{1-ae^{i\omega t}}$  is periodic:  $f(t+T) = f(t)$ . Find  $T$ .

17. Compute the Fourier expansion of the function  $f(t) = \frac{1}{1-ae^{i\omega t}}$  ( $|a| < 1$ ). Find the power spectrum of this function.

18.  $z = 3 + 4i$ . Write the complex number  $z$  in (a) polar form  $z = re^{i\theta}$ ; (b) in the form of a  $2 \times 2$  matrix.

19. Fill in the blanks:

Ability	C	C++	Python	FORTRAN	Maple	Mathematica	MatLab	L <sup>A</sup> T <sub>E</sub> X
Clueless								
Maybe								
Fluent								

20. Where are you weak? What do you (think you) need to learn?

Linear Algebra	Eigenstuff	
Vector Fields	Vector Operators	
Geometry/Trigonometry	Computer aids for calculations	
Special Functions	Complex Variables	
Writing	Fourier Stuff	
How to find webstuff	How to evaluate webstuff	
Probability/Statistics	Other (specify)	