1. 10 particles of equal mass $m$ are connected by 11 identical springs with equilibrium length $l$ and spring constant $k$ to each other and to brick walls at the two ends, in the spirit of Problem Set #2, Problem 2.

   a. Draw a picture.

   b. Compute the oscillation frequencies by diagonalizing the appropriate matrix (assume $k = 1, m = 1$).

   c. Plot the eigenfrequencies.

   d. Plot the three “lowest eigenvectors”, as described in class.

   e. Do these curves remind you of something? What?

   f. Test your guess by plotting your guess against these eigenvectors.

2. Solve the heat flow equation on a ring of radius $a$:

   $$C\rho \frac{\partial T(\theta, t)}{\partial t} = \frac{\kappa}{a^2} \frac{\partial^2 T(\theta, t)}{\partial \theta^2}$$

   for initial conditions $T(\theta, t = 0) = 100|\theta|/\pi$.

3. Use Maple (or other) to compute the Fourier series expansion of the function

   $$f(\theta) = \begin{cases} \theta(\pi - \theta) & \text{for } 0 \leq \theta \leq \pi \\ -f(-\theta) & \text{for } -\pi \leq \theta < 0 \end{cases}$$

4. $f(x) = x^2$. 
a. Integrate \( f(x)^2 \) from \(-\pi\) to \(+\pi\).

b. “Compute” (i.e., you can look it up at 24.14) the Fourier series expansion of \( f(x) \).

c. Square the fourier expansion and then integrate this mess from \(-\pi\) to \(+\pi\).
   (Hint: look at Parseval’s Identity, 24.5.)

d. Construct an expression for \( \sum_{k=1}^{\infty} \frac{1}{k^4} \).

e. Compare this result with 21.20.